1.71 Bestäm $f'(0)$ om $f(x) = \arcsin x$

Lektion 9X

Lösning:

$$y = f(x) = \arcsin x$$

$$x = \boxed{f^{-1}(y) = \sin y} = \sin \arcsin x$$

$$\boxed{D(f(x)) = \frac{1}{D(f^{-1}(y))}}$$

$$D(f(x)) = \frac{1}{D(f^{-1}(y))} = \frac{1}{\cos y} = \left[\begin{array}{l} x = \sin y \\ \text{Trig-ettan} \\ \cos^2 y + \sin^2 y = 1 \end{array}\right] = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}$$

$$\cos y = \sqrt{1-\sin^2 y}$$

$$f'(0) = \frac{1}{\sqrt{1-0^2}} = 1$$

Svar:
$f'(0) = 1$

1.73 Bestäm ekvationen för tangenten till funktionen $f(x) = \arcsin x$ i punkten $\left(\frac{1}{2}, \frac{\pi}{6}\right)$.

Lösning: 1) Söker efter k-värde $\Rightarrow f'\left(\frac{1}{2}\right)$

2) Tangentens ekv kan bestämmas med enpunktform $(y - y_1) = k(x - x_1)$

1)
$$y = f(x) = \arcsin x$$
$$x = f^{-1}(y) = \sin y$$

$$f'(x) = \frac{1}{D\left(f^{-1}(y)\right)} = \ldots = \frac{1}{\sqrt{1 - x^2}}$$

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$$f'\left(\frac{1}{2}\right) = \frac{1}{\sqrt{1 - \left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{3}}$$, k-värdet.

2) $$\left(y - \frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}\left(x - \frac{1}{2}\right)$$

Svar: $y = \frac{2}{\sqrt{3}}x + \frac{\pi}{6} - \frac{1}{\sqrt{3}}$

1.74 Bestäm derivatan av

a) $f(x) = \arcsin(2x+1)$

Lösn: $y = f(x) = \arcsin(2x+1)$

$$\sin y = \sin f(x) = \sin \arcsin(2x+1) = 2x+1$$

$$x = f^{-1}(y) = \frac{\sin y - 1}{2}, \quad f'(x) = \frac{1}{D\left(f^{-1}(y)\right)} = \frac{2}{\cos y} = \frac{2}{\sqrt{1-\sin^2 y}} =$$

$$= \frac{2}{\sqrt{1-(2x+1)^2}} = \frac{\cancel{2}}{\sqrt{\cancel{1}-\cancel{4}x^2-\cancel{4}x-\cancel{1}}} = \frac{1}{\sqrt{-x^2-x}}$$

1.74

b) Derivera $f(x) = \arcsin x^2$

Lösning: $y = f(x) = \arcsin x^2$

$$\sin y = \sin \arcsin x^2 = x^2$$

$$x = f^{-1}(y) = \pm\sqrt{\sin y} = \sqrt{\sin y}, \quad 0 \leq y < \pi/2$$

$$f'(x) = \frac{1}{D\left(f^{-1}(y)\right)} = \frac{1}{\frac{1}{2}(\sin y)^{-1/2} \cdot \underbrace{\cos y}_{\sqrt{1-\sin^2 y}}} = \frac{2}{\frac{1}{x} \cdot \sqrt{1-x^4}} = \frac{2x}{\sqrt{1-x^4}}$$

1.74 c) $y = f(x) = \arcsin\sqrt{x}$

$$\sin y = \sin \arcsin \sqrt{x} = \sqrt{x}$$

$$x = f^{-1}(y) = \sin^2 y$$

$$f'(x) = \frac{1}{D(f^{-1}(y))} = \frac{1}{2\sin y \cdot \cos y} = \frac{1}{2\sqrt{x} \cdot \sqrt{1-x}} = \frac{1}{2\sqrt{x - x^2}}$$

d) $f(x) = x \cdot \arcsin x$

$y = f \cdot g \ , \ y' = f'g + fg'$

Lösn: $f'(x) = \arcsin x + x \cdot D(\arcsin x)$

$$f'(x) = \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}}$$

1.75 Låt $f(x) = \arccos x$

a) Visa att $f'(x) = -\dfrac{1}{\sqrt{1-x^2}}$

Lösn: $y = f(x) = \arccos x$

$x = f^{-1}(y) = \cos y$

$$D(f(x)) = \frac{1}{D(f^{-1}(y))} = \frac{1}{-\sin y} = \left[\begin{array}{l}\text{Trig. ettan.} \\ \cos^2 y + \sin^2 y = 1 \\ \sin y = \sqrt{1-\cos^2 y}\end{array}\right] = \frac{1}{-\sqrt{1-\cos^2 y}} = \frac{-1}{\sqrt{1-x^2}}$$

b) Beräkna $f'(0) = \dfrac{-1}{\sqrt{1-0^2}} = -1$