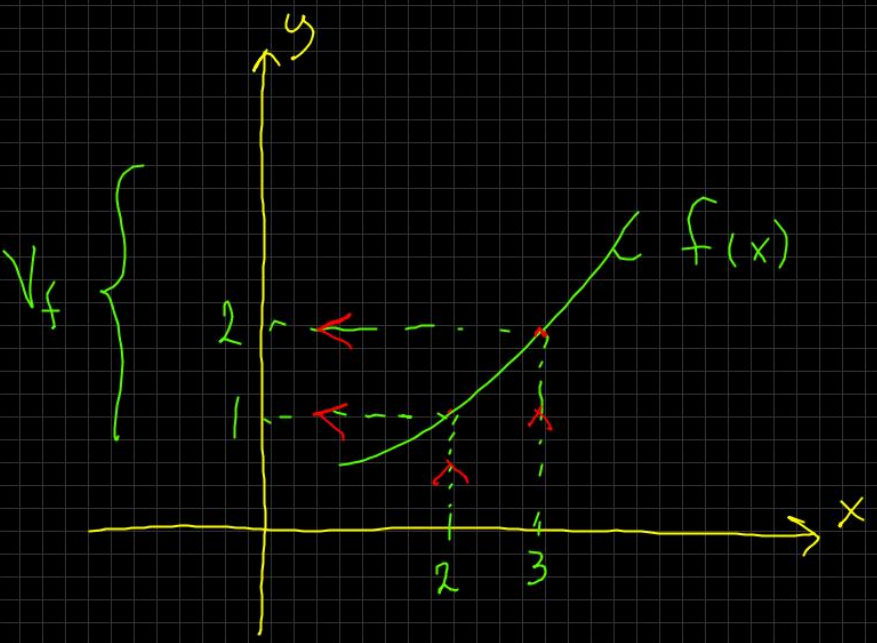
"välkammad", för varje x finns det unikt y

heltäckande,

- injektiv
- surjektiv
- bijektiv

1-1

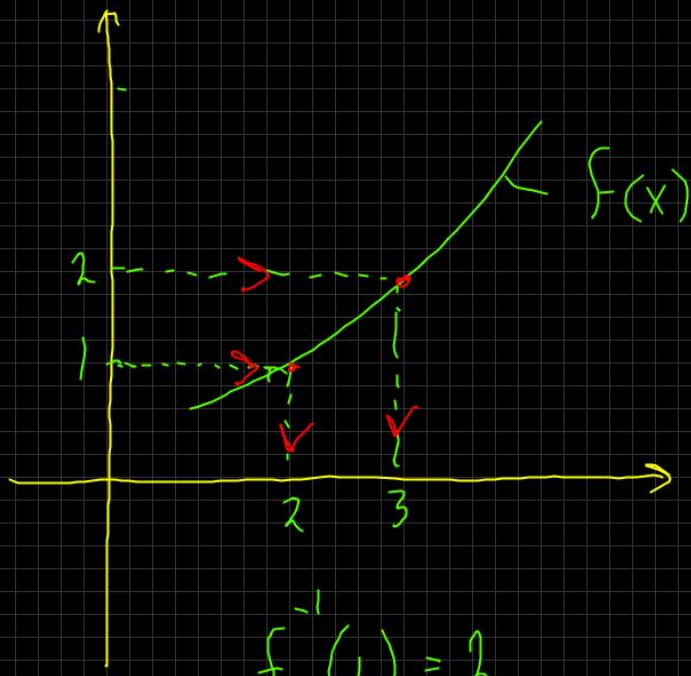
Invers funktion



$f(2) = 1$

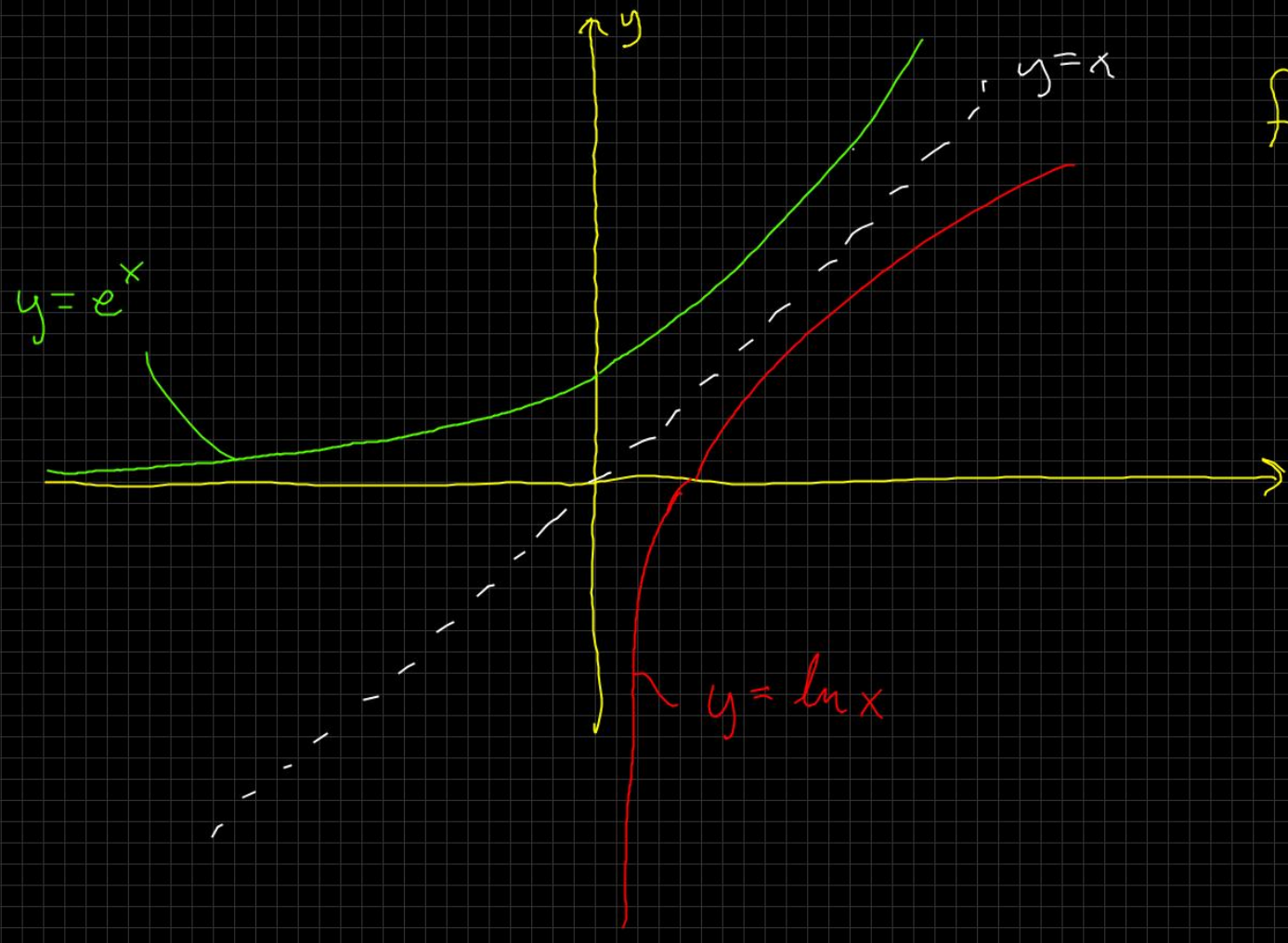
$f(3) = 2$

Inverse function ?



$f^{-1}(1) = 2$

$f^{-1}(2) = 3$



$$f(x) = e^x \quad , x \in \mathbb{R}$$

$$f^{-1}(f(x)) = x$$

$$f^{-1}(e^x) = x$$

$$\ln(e^x) = x$$

$$f^{-1}(x) = \ln x$$

Ex 1) $f(x) = 2x - 6$. Bestäm $f^{-1}(5)$

Bestäm $f^{-1}(x)$

$f^{-1} = g(x)$

$$\begin{cases} f(g(x)) = 2 \cdot g(x) - 6 \\ f(g(x)) = x \end{cases}$$

$$2\,g(x) - 6 = x$$

$$2\,g(x) = x + 6$$

$$g(x) = \frac{x+6}{2}$$, invers fn. f^{-1}

$$f(g(x)) = g(f(x)) = x$$

$$f^{-1}(5) = \frac{5+6}{2} = 5{,}5$$

Ex 2)

$$f(g(x)) = e^{\ln x} = x$$

$$f(g(x)) = \cos(\arccos x) = x$$

$$\sin(\arcsin x) = x$$

$$g(f(x)) = \ln e^{x} = x$$

$$g(f(x)) = \arccos(\cos x) = x$$

$$\arcsin(\sin x) = x$$

$f(x)$		$g(x)$
e^{x}	$\longleftrightarrow$	$\ln x$
$\cos x$	$\longleftrightarrow$	$\arccos x$
$\sin x$	$\longleftrightarrow$	$\arcsin x$

$$f(x) = 2x - 6$$

$$f^{-1} = g(x)$$

$$f(g(x)) = 2g(x) - 6 = x$$

$$g(x) = \frac{x+6}{2}$$

1.44 d)

$$f(x) = \frac{x^3+1}{x} = x^2 + \frac{1}{x}$$

$$x^3 + 0 + 0 + 1 \;\big|\; x$$
$$-x^3 \qquad\quad x^2$$
$$\overline{\qquad 0 \qquad}$$

$$f' = \frac{3x^2 \cdot x - (x^3+1)\cdot 1}{x^2} = \frac{2x^3 - 1}{x^2}$$

$$(x^{-1})' = -\frac{1}{x^2}$$

$$f' = 0$$

$$2x = \frac{1}{x^2}$$

$$x^3 = \frac{1}{2}$$

$$x = \sqrt[3]{1/2} \quad \text{int. è } f$$

$$f\left(\sqrt[3]{1/2}\right) = \frac{\left(\sqrt[3]{1/2}\right)^3 + 1}{\sqrt[3]{1/2}} = \frac{3/2}{\sqrt[3]{1/2}} =$$

$$= \frac{3}{2\sqrt[3]{1/2}} = \frac{3}{\sqrt[3]{2^3}\cdot\sqrt[3]{1/2}} = \frac{3}{\sqrt[3]{2^3 \cdot 1/2}} = \frac{3}{\sqrt[3]{4}}$$