Lektion 6. Räknepass

Kedjeregel	$y = f(g(x))$	$y' = f' \cdot g'$
Produktregel	$y = f \cdot g$	$y = f'g + fg'$
Kvotregel	$y = \dfrac{f}{g}$	$y = \dfrac{f'g - fg'}{g^2}$

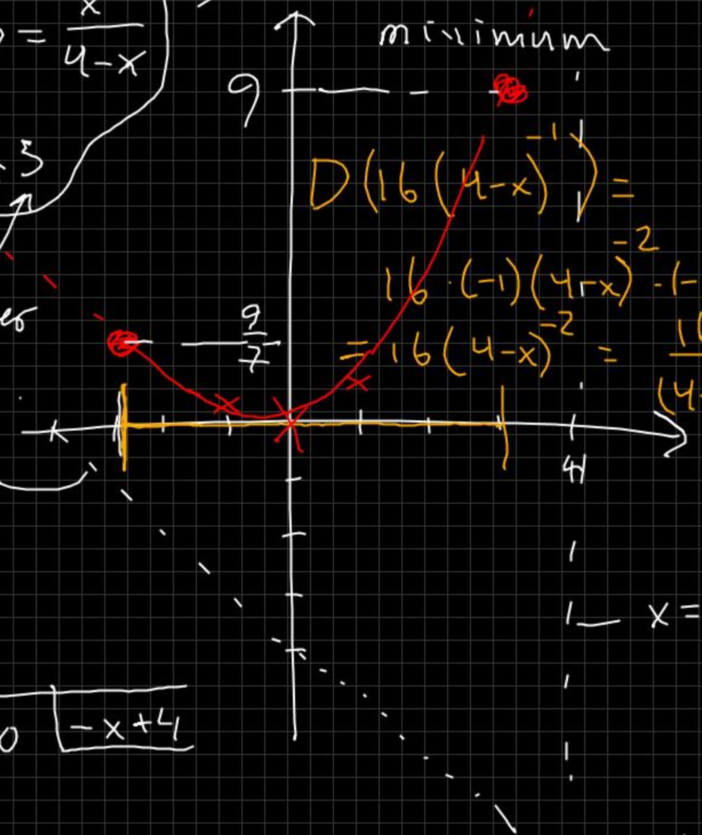
Ex: $f(x) = \dfrac{x^2}{4-x}$

$-3 \leq x \leq 3$

ändpunkter

maximum

minimum



$D\left(16(4-x)^{-1}\right) =$

$16 \cdot (-1)(4-x)^{-2} \cdot (-1)$

$= 16(4-x)^{-2} = \dfrac{16}{(4-x)^2}$

asymptot

$y = -x-4$

$$
\begin{array}{l}
-x - 4 \\
\hline
x^2 + 0 + 0 \quad \boxed{-x+4} \\
-(x^2 - 4x) \\
\hline
0 + 4x + 0 \\
-(+4x - 16) \\
\hline
0 + 16
\end{array}
$$

$f(x) = -x - 4 + \dfrac{16}{4-x}$

1.43 Ange största värde för funktionen

$$f(x) = \frac{x^2}{4-x}, \quad -3 \leq x \leq 3.$$

Lektion	Grundkurs	Extra
	Grundläggande begrepp	
1,2	1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.11	
	Funktioner	
1,2	1.12, 1.13, 1.14, 1.15, 1.16	
	Polynomfunktioner	
1,2	1.17, 1.18	
	Kontinuitet	
3	1.20, 1.21, 1.22, 1.23, 1.24, 1.25	
	Deriverbarhet och absolutbelopp	
4	1.26, 1.27, 1.28, 1.29, 1.30, 1.31, 1.34, 1.35	*1.32, 1.33, 1.36, 1.37*
	Rationella funktioner	
5, 6	(1.38, 1.39), 1.40, 1.41, 1.42, 1.44, 1.45	*1.46, 1.47*

Lösning:

$$f(x) = -x - 4 + \frac{16}{4-x} \quad \text{(se föregående sida)}$$

Oscar räknar idag

1.41 f, g, h, i, j

$$f(x) = -x - 4 + 16(4-x)^{-1}$$

$$\begin{cases} f'(x) = -1 + 0 + 16\cdot\cancel{(-1)}\cdot(4-x)^{-2}\cdot\cancel{(-1)} = -1 + \dfrac{16}{(4-x)^2} \\ f'(x) = 0 \end{cases}$$

√ 1.43

$$\frac{16}{(4-x)^2} = 1 \qquad 16 = (4-x)^2 \qquad 4-x = \pm 4$$

$x_1 = 0 \Rightarrow f(0) = 0$ minimum

$x_2 = 8$

1.43

$$f(x) = \frac{x^2}{4-x}, \quad -3 \leq x \leq 3$$

$$f(-3) = \frac{(-3)^2}{4-(-3)} = \frac{9}{7}$$

$$f(3) = \frac{3^2}{4-3} = 9$$ (, största värdet.)

Svar: $f(x)$ har största värdet 9 då $x = 3$.

1.41

f) $f(x) = \dfrac{2x^2-3x+3}{x-1} = 2x-1+\dfrac{2}{x-1}$

Asymptoter $\begin{cases} |x| \to \infty, & y = 2x-1 \\ \text{ej def}, & x = 1 \end{cases}$

$$\begin{array}{l|l} \quad 2x \quad - 1 & \\ \hline 2x^2 - 3x + 3 & x-1 \\ -(2x^2 - 2x) & \\ \hline 0 \quad - x + 3 & \\ \quad -(-x + 1) & \\ \hline \quad\quad 0 + 2 & \end{array}$$

g) $f(x) = \dfrac{6+4x-x^2}{x+1} = -x+5+\dfrac{1}{x+1}$

Asymptoter $\begin{cases} |x| \to \infty, & y = -x+5 \\ \text{ej def}, & x = -1 \end{cases}$

$$\begin{array}{l|l} -x \quad + 5 & \\ \hline -x^2 + 4x + 6 & x+1 \\ -(-x^2 - x) & \\ \hline 0 \quad + 5x + 6 & \\ \quad -(5x + 5) & \\ \hline \quad\quad 0 + 1 & \end{array}$$

h) $f(x) = \dfrac{2x^3 - 3x^2 - x + 3}{x^2 - 1} = 2x - 3 + \boxed{\dfrac{x}{x^2 - 1}}$

$$\begin{array}{l|l} 2x^3 - 3x^2 - x + 3 & x^2 - 1 \\ \hline -(2x^3 \quad\quad - 2x) & 2x - 3 \\ \hline 0 - 3x^2 + x + 3 & \\ -(-3x^2 \quad\quad + 3) & \\ \hline 0 + x + 0 & \end{array}$$

$$\frac{x}{x^2 - 1} = \frac{\frac{x}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \frac{\frac{1}{x}}{1 - \frac{1}{x^2}}$$

$|x| \to \infty, \; y = 0$

Svar

$f(x): \begin{cases} |x| \to \infty, \; y = 2x - 3 + 0 \\ \text{ei def.} \quad x = \pm 1 \end{cases}$

i) $$f(x) = \frac{3x^3 - 4x^2 + 2}{x^2+1} =$$

$$\begin{array}{l|l} 3x^3 - 4x^2 + 0 + 2 & x^2+1 \\ \hline -(3x^3 \quad + 3x) & 3x - 4 \\ 0 - 4x^2 - 3x + 2 & \\ -(-4x^2 \quad - 4) & \\ 0 \quad -3x + 6 & \end{array}$$

$$f(x) = 3x - 4 - \boxed{\frac{3x-6}{x^2+1}}$$

$$\frac{3x-6}{x^2+1} = \frac{\frac{3x}{x^2} - \frac{6}{x^2}}{\frac{x^2}{x^2} - \frac{1}{x^2}} = \frac{\frac{3}{x} - \frac{6}{x^2}}{1 - \frac{1}{x^2}} \Rightarrow |x| \to \infty,\ y = 0$$

$f(x)$ asymptoter $|x| \to \infty,\ y = 3x - 4$

Ingen lodrät asymptot

ty, $x^2+1 > 0,\ \forall x \in \mathbb{R}$

j)

$$f(x) = \frac{x^3 - 5x^2 + 6x - 2}{x^2 - 4x + 3}$$

$$f(x) = x - 1 - \frac{x-1}{x^2 - 4x + 3}$$

Asymptoter $\begin{cases} |x| \to \infty, & y = x - 1 \\ \text{ei def.} & x = 1,\ x = 3 \end{cases}$

$$x^2 - 4x + 3 = (x-1)(x-3)$$

$$\begin{array}{l|l} \multicolumn{2}{l}{x - 1} \\ \hline x^3 - 5x^2 + 6x - 2 & x^2 - 4x + 3 \\ -(x^3 - 4x^2 + 3x) & \\ \hline 0 - x^2 + 3x - 2 & \\ -(-x^2 + 4x - 3) & \\ \hline 0 - x + 1 = 1 - x & \end{array}$$

$$+ \frac{1-x}{x^2 - 4x + 3}$$

$$-(x - 1)$$