Rationella funktioner

$$f(x) = \frac{p(x)}{q(x)}$$

Fall:

I grad $p(x) < $ grad $q(x)$

II grad $p(x) = $ grad $q(x)$

III grad $p(x) > $ grad $q(x)$

Ex) $$f(x) = \frac{2x^2 + x}{x^3 + 1}$$

Ex) $$f(x) = \frac{2x + 1}{3x + 5}$$

Ex) $$f(x) = \frac{x + 3}{2}$$

Asymptoter (rät)

$$f(x) = \frac{p(x)}{q(x)}$$

Fall:

I grad $p(x) < $ grad $q(x)$

II grad $p(x) = $ grad $q(x)$

III grad $p(x) > $ grad $q(x)$

I $$f(x) = \frac{\frac{2x^2}{x^3} + \frac{x}{x^3}}{\frac{x^3}{x^3} + \frac{1}{x^3}} = \frac{\frac{2}{x} + \frac{1}{x^2}}{1 + \frac{1}{x^3}}$$

Dividera alla termer med den högsta graden

$|x| \to \infty \to \begin{cases} y = 0 \\ x = -1 \end{cases}$ ej definierad

II $$f(x) = \frac{2x+1}{3x+5} = \frac{2}{3} + \frac{-\frac{7}{3}}{3x+5}$$

Polynomdivision

$$\begin{array}{l|l} \frac{2}{3} & \\ \hline 2x+1 & 3x+5 \\ -\left(2x+\frac{10}{3}\right) & \\ \hline 0 - \frac{7}{3} & \end{array}$$

$|x| \to \infty$ $\quad y = \frac{2}{3}$

ej def. för $\quad x = -5/3$

III

$$f(x) = \frac{x+3}{2} = \frac{1}{2}x + \frac{3}{2}$$

grad $p(x)$ > grad $q(x)$

$$\begin{array}{l|l} \multicolumn{1}{l}{\frac{1}{2}x + \frac{3}{2}} & \\ \hline x + 3 & 2 \\ \cline{2-2} -(x) & \\ \cline{1-1} 0 + 3 & \\ -(3) & \\ \hline 0 & \end{array}$$

Vad händer då:

$|x| \to \infty$, $y = \frac{1}{2}x + \frac{3}{2}$

141) Vilka är asymptoterna till följande rationella uttryck

a) $f(x) = \dfrac{x}{x+1} =$ Fall II

$$\begin{array}{l|l} \multicolumn{2}{c}{1} \\ \hline x & x+1 \\ -(x+1) & \\ \hline 0 \;\; -1 & \end{array}$$

$= 1 - \dfrac{1}{x+1}$

Svar: $|x| \to \infty,\ y = 1$

ej def. $x = -1$

b) $f(x) = \dfrac{6x^2}{1-4x^2} =$

$$\begin{array}{l|l} \multicolumn{2}{c}{-\frac{3}{2}} \\ \hline 6x^2 + 0 + 0 & -4x^2 + 1 \\ -\left(6x^2 \quad - \frac{3}{2}\right) & \\ \hline 0 \quad + \frac{3}{2} & \end{array}$$

$= -\dfrac{3}{2} + \dfrac{3/2}{1-4x^2}$

$|x| \to \infty,\ y = -\dfrac{3}{2}$

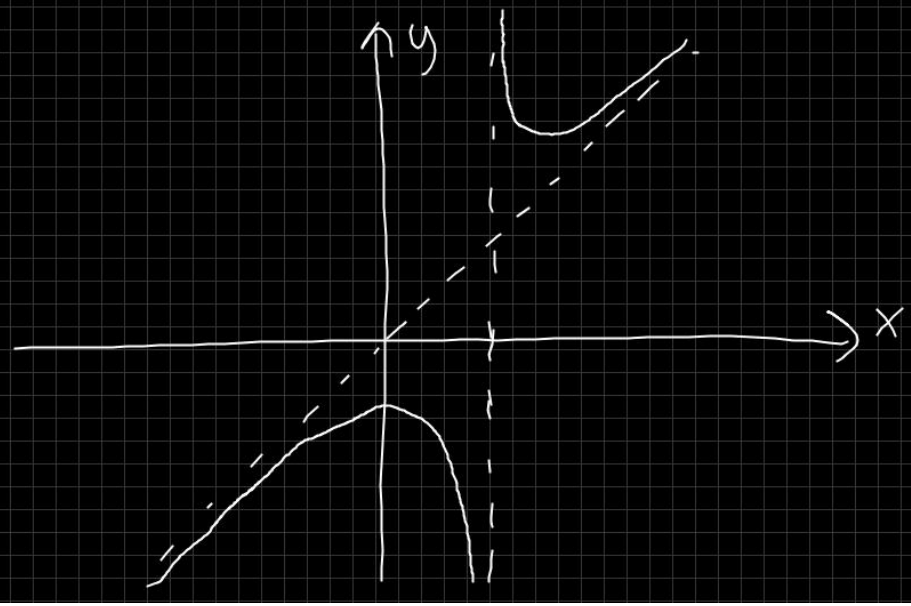
ej.def. $x = \pm\dfrac{1}{2}$

c) $f(x) = \dfrac{x^2 - x + 1}{x-1} = x + \dfrac{1}{x-1}$

$$
\begin{array}{r|l}
x^2 - x + 1 & x-1 \\
-(x^2 - x) & x \\
\hline
0 + 0 + 1 &
\end{array}
$$

$|x| \to \infty$, $y = x$

ei def. $x = 1$



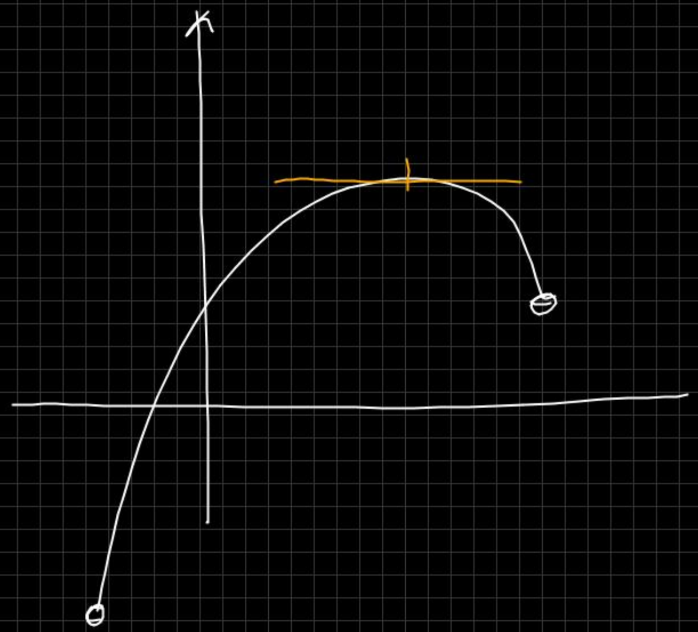
Deriveringsregler:

Kedjeregel $\quad y = f(g(x)) \longrightarrow y' = \boxed{f' \cdot g'}$

Produktregel $\quad f = u \cdot v \longrightarrow f' = \boxed{u'v + u \cdot v'}$

Kvotregel $\quad f = \dfrac{u}{v} = u \cdot v^{-1} \longrightarrow f' = u' \cdot v^{-1} + u \cdot (-1) \cdot v^{-2} \cdot v'$

$$= \frac{u'}{v} \cdot \frac{v}{v} - \frac{u \cdot v'}{v^2} = \boxed{\frac{u'v - uv'}{v^2}}$$



141

d) $f(x) = \dfrac{(x+1)^2}{x} = \dfrac{x^2+2x+1}{x} = \left[\begin{array}{l|l} \multicolumn{2}{l}{x+2} \\ \hline x^2+2x+1 & x \\ -(x^2) & \\ \hline 0+2x & \\ \quad -(2x) & \\ \hline \quad 0+1 & \end{array}\right] = x+2+\dfrac{1}{x}$

$|x| \to \infty,\ y = x+2$

ei def, $x = 0$

e) $f(x) = \dfrac{x(x-4)}{2(2-x)} = \dfrac{x^2-4x}{4-2x} = \left[\begin{array}{l|l} \multicolumn{2}{l}{-\frac{1}{2}x+1} \\ \hline x^2-4x & -2x+4 \\ -(x^2-2x) & \\ \hline 0-2x+0 & \\ \quad -(-2x+4) & \\ \hline \quad 0-4 & \end{array}\right] = -\dfrac{1}{2}x+1-\dfrac{4}{4-2x}$

$|x| \to \infty,\ y = \dfrac{x}{2}+1$

ei def, $x = 2$