2.78 Beräkna $\displaystyle\int_0^1 \left(\frac{x}{3} - \frac{x^2}{5}\right) dx = \left[\frac{x^2}{6} - \frac{x^3}{15}\right]_0^1 =$

Övningar 1

2.78 ✓

2.97

$$\left(\frac{1^2}{6} - \frac{1^3}{15}\right) - \left(0 - 0\right) = \frac{1 \cdot 5}{2 \cdot 3 \cdot 5} - \frac{1 \cdot 2}{3 \cdot 5 \cdot 2} = \frac{5-2}{30} = \frac{1}{10}$$

2.97 *

a) $$\int \frac{1+2x}{1+x^2}\,dx = \underbrace{\int \frac{1}{1+x^2}\,dx}_{\arctan x} + \underbrace{\boxed{\int \frac{2x}{1+x^2}\,dx}}_{I}$$

partiell int.

$$I = \int 2x \cdot \frac{1}{1+x^2}\,dx = 2x \cdot \arctan x - \boxed{\int 2 \arctan x\,dx}$$

$$\int 2 \cdot \arctan x\,dx = 2x \cdot \arctan x - \underbrace{\boxed{\int 2x\,\frac{1}{1+x^2}\,dx}}_{I}$$

partiell integration

$$\boxed{I = \int \frac{2x}{1+x^2}\, dx} = \cancel{2x \arctan x} - \cancel{2x \arctan x} + \int \frac{2x}{1+x^2}\, dx$$

Summeret:

$$\int \ldots \, dx = \arctan x + 2x \arctan x$$

2,97 *

a) $$\int \frac{1+2x}{1+x^2}\,dx = \cancel{\left[\begin{array}{ll} t = x^2 & x = t^{1/2} \\ \frac{dx}{dt} = \frac{1}{2} t^{-1/2} & dx = \frac{1}{2} t^{-1/2}\,dt \end{array}\right]} =$$

$$\cancel{\int \frac{(1+2t^{1/2})}{(1+t)} \cdot \frac{1}{2} \cdot t^{-1/2}\,dt} \;\cancel{= \frac{1}{2}\int \frac{t^{-1/2}}{1+t}\,dt + \int \frac{1}{1+t}\,dt =}$$

HÄR

$$\int \frac{1}{1+x^2}\,dx + \int \frac{\overset{\cancel{2t^{1/2}}}{(2x)}}{\underset{1+t}{(1+x^2)}}\,\underset{\text{um}}{dx}\,\cancel{\frac{1}{2} t^{-1/2}}\,\overline{dt}$$

Varsdal subst.

$$\left[\begin{array}{ll} t = x^2 & dt = 2x\,dx \\ \frac{dt}{dx} = 2x & \end{array}\right] \quad \left[\begin{array}{ll} t = x^1 & x = t^{1/2} \\ \frac{dx}{dt} = \frac{1}{2} t^{-1/2} & dx = \frac{1}{2} t^{-1/2}\,dt \end{array}\right]$$

$$= \arctan x + \int \frac{1}{1+t}\,dt = \arctan x + \ln(1+t) = \arctan x + \ln(1+x^2)$$