1.103 Vilken D_f och vilken V_f har $f(g(x))$ då

1.35 c

$$f(x) = 1 + \sqrt{x+4}$$

$$g(x) = x^2 + 4x$$

Lösning:

· Yttre fn. f. Vilken def.m är tillåtna ⇒ värdem. för g.

$\sqrt{z} \in \mathbb{R}^+ \Rightarrow z \in \mathbb{R}^+ \Rightarrow x+4 \geq 0$

$x \geq -4$ ← Def.m för f(x)

⇒ Värdem. för g(x)!

$\boxed{g(x)} = x^2 + 4x$

$g(x) \geq -4$

· Vad är g(x) minimum?

$$\begin{cases} g'(x) = 2x+4 \\ g'(x) = 0 \end{cases}$$

$x = -2$ minimum $g(-2) = (-2)^2 + 4\cdot(-2) = -4$

∴ $x \in \mathbb{R} = D_f$

$y \geq 1 \quad = V_f$

1.110 a) Visa att $f(x) = \dfrac{x^2+3}{x+1}$ kan skrives $f(x) = x - 1 + \dfrac{4}{x+1}$

Lösning:

$$
\begin{array}{l|l}
\quad x - 1 & \\
\hline
x^2 + 0x + 3 & x + 1 \\
-(x^2 + x) & \\
\hline
0 - x + 3 & \\
-(-x - 1) & \\
\hline
\quad 0 \quad 4 &
\end{array}
$$

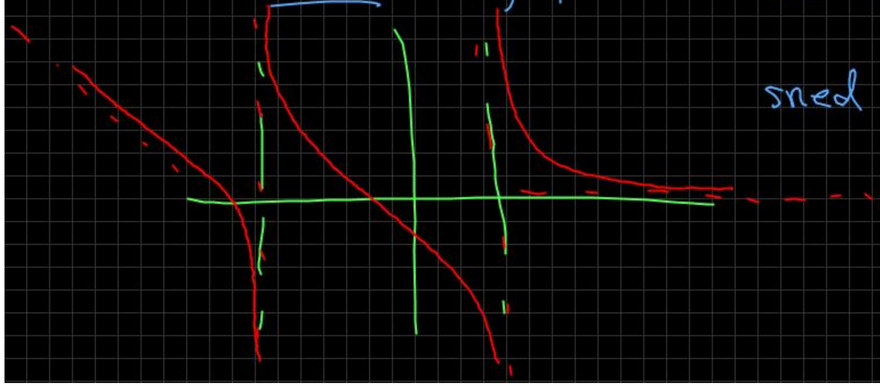
restterm

$$f(x) = x - 1 + \frac{4}{x+1}$$

b) Vilka asymptoter har $f(x)$?

Svar: asymptot: lodrät $x = -1$ (nämnaren $= 0$)

sned $|x| \to \infty$ $y = x - 1$



1.35 c) För vilka x är

$$|x^2-9| = |x+2|+5 \; ?$$

Lösning:

$$|(x+3)(x-3)| = |x+2|+5$$

① $(x+3)(x-3) = -(x+2)+5 \quad , \quad x < -3$

② $-\big((x+3)(x-3)\big) = -(x+2)+5 \quad , \quad -3 \leq x < -2$

③ $-\big((x+3)(x-3)\big) = x+2+5 \quad , \quad -2 \leq x < 3$

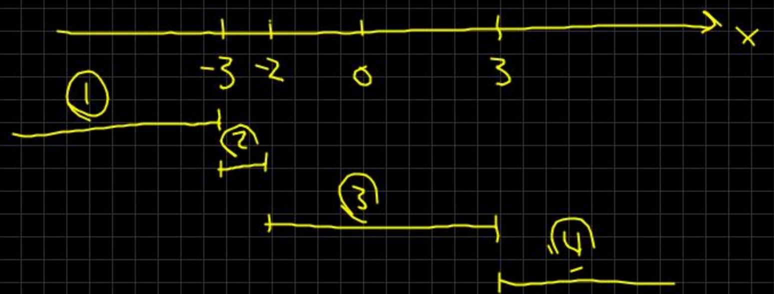
④ $(x+3)(x-3) = x+2+5 \quad , \quad x \geq 3$

intressant teckenbyten:

$x = -3$

$x = 3$

$x = -2$



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$$x^2 - 9 = -(x+2) + 5$$

$$x^2 + x - 12 = 0$$

$$x = -\frac{1}{2} \pm \sqrt{\left(\frac{1}{2}\right)^2 + 12}$$

$$x = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{48}{4}}$$

$$x = -\frac{1}{2} \pm \frac{7}{2}$$

$$\cancel{x_1 = 3}$$

$$x_2 = -4$$

kontroll mot intervall $x < -3$

1.122

$$f(x) = \sqrt[3]{x+1} = (x+1)^{1/3}$$

$$f(0) = 1 \Rightarrow a_0 = 1$$

$$f'(x) = \frac{1}{3}(x+1)^{-2/3}, \quad f'(0) = \frac{1}{3} \Rightarrow a_1 = \frac{1}{3}$$

$$f''(x) = -\frac{2}{9}(x+1)^{-5/3}, \quad f''(0) = -\frac{2}{9} \Rightarrow a_2 = \frac{-\frac{2}{9}}{2!} = -\frac{1}{9}$$

$$f^{(3)}(x) = \frac{10}{27}(x+1)^{-8/3}, \quad f^{(3)}(0) = \frac{10}{27} \Rightarrow a_3 = \frac{\frac{10}{27}}{3!} = \frac{5}{81}$$

$$f^{(4)}(x) = -\frac{80}{81}(x+1)^{-11/3}, \quad f^{(4)}(0) = -\frac{80}{81} \Rightarrow a_4 = \frac{-\frac{80}{81}}{4!} = -\frac{5}{243}$$

$$f(x) \approx f(0) + f'(0)\cdot x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \cdots$$

$$p(x) \approx \boxed{a_0 + a_1 x + a_2 x^2 + \ldots} + \frac{f^{(4)}(0)}{4!}x^4 \ldots$$

$\approx$ felet

$$f(x) \approx 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 \boxed{- \frac{5}{243}x^4 + \ldots}$$

gränsvärdet?

173 Bestäm tangenten ($y = kx + m$) till $f(x) = \arcsin x$ i punkten $\left(\frac{1}{2}, \frac{\pi}{6}\right)$

Lösning:

$$k = f' = D(f(x))$$

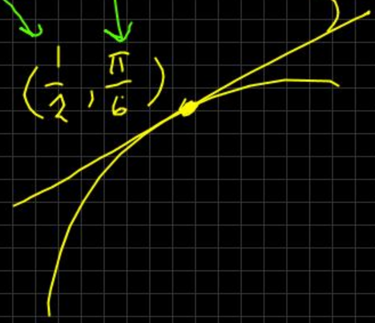
$$D\big(f(x)\big) = \frac{1}{D\big(f^{-1}(y)\big)} = \frac{1}{\quad}$$

$$\boxed{\begin{aligned} y - y_1 &= k(x - x_1) \\ y - \frac{\pi}{6} &= \frac{\sqrt{3}}{2}\left(x - \frac{1}{2}\right) \\ y &= \frac{\sqrt{3}}{2}x - \frac{\sqrt{3}}{4} + \frac{\pi}{6} \end{aligned}}$$

x y

$\left(\frac{1}{2}, \frac{\pi}{6}\right)$

Lutning $k = f'\left(\frac{1}{2}\right)$



$$y = f(x) = \arcsin x$$

$$\sin y = \sin(\arcsin x) = x = f^{-1}(y) = \sin y$$

$$D\big(f^{-1}(y)\big) = \cos y$$

$$D\left(f^{-1}\left(\frac{\pi}{6}\right)\right) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} = k$$

Svar: