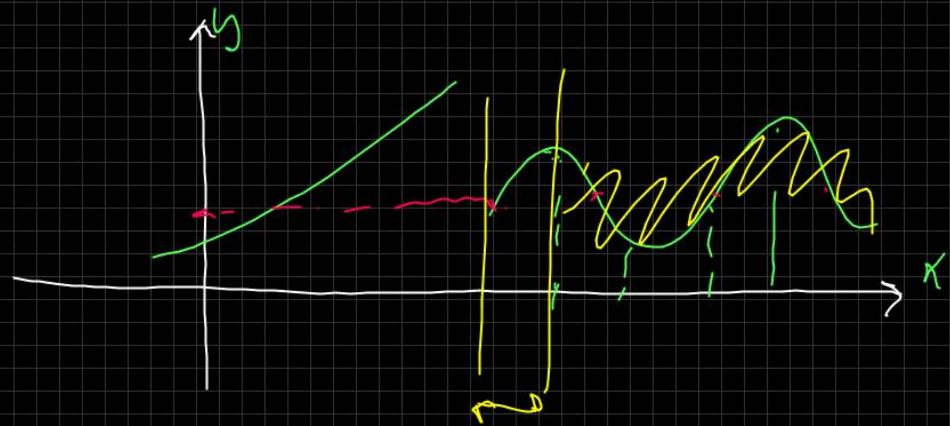
- Derivatan av invers
- Invers till icke injektiv funktion

REPETITION

kap. 1
kap. 2



$y = \arccos x \qquad y' = \frac{-1}{\sqrt{1-x^2}}$

Derivata av invers:

$$\boxed{D\left(f(x)\right) = \frac{1}{D\left(f^{-1}(y)\right)}}$$

Deriver $\quad y' = \frac{-1}{\sqrt{\frac{1}{4} - x^2}}$

$y = f(x) = \arccos 2x \qquad z = 2x$

$y' = \frac{dy}{dz}\frac{dz}{dx} = y' = D(\arccos z)\cdot 2$

$\cos y = \cos(\arccos 2x) = 2x$

$\frac{-2}{\sqrt{1-(2x)^2}} = \qquad \sin^2 y + \cos^2 y = 1$

$\sin y = \sqrt{1-\cos^2 y}$

$f^{-1}(y) = \frac{1}{2}\cos y = x$

$D\left(f^{-1}(y)\right) = -\frac{1}{2}\sin y = -\frac{1}{2}\sqrt{1-\cos^2 y} = -\frac{1}{2}\sqrt{1-(2x)^2} = \frac{1}{2}\sqrt{1-4x^2}$

$= -\frac{1}{2}\sqrt{4\left(\frac{1}{4}-x^2\right)} = -\sqrt{\frac{1}{4}-x^2}$

$$\frac{-2}{\sqrt{1-(2x)^2}} = \boxed{\frac{-2}{\sqrt{1-4x^2}}} = \frac{-2}{\sqrt{4\left(\frac{1}{4}-x^2\right)}}$$

$$= \frac{-\cancel{2}}{\cancel{2}\sqrt{\frac{1}{4}-x^2}} = \frac{-1}{\sqrt{\frac{1}{4}-x^2}}$$

- Icke injektiv.

$\Rightarrow$ Begränsa intervallet i x-led

$\Rightarrow$ begränsad injektiv funktion.

$\Rightarrow$ invertera.

$$y = f(x) = \arcsin(2x+1)$$

$$\cancel{\sin y} = \overbrace{\sin(\arcsin(2x+1))}^{VL} = \overbrace{2x+1}^{HL}$$

$$\cos(\qquad)\cdot(\arcsin(2x+1))' = 2$$

$$(\arcsin(2x+1))' = \frac{2}{\cos(\arcsin(2x+1))}$$

$$(\arcsin(2x+1))' = ?$$

$$\boxed{\cos y = \sqrt{1-\sin^2 y}}$$

$$\left(\arcsin(2x+1)\right)' = \frac{2}{\sqrt{1-\sin^2(\arcsin(2x+1))}}$$

$$= \frac{2}{\sqrt{1-(2x+1)^2}} = \frac{2}{\sqrt{\cancel{1}-4x^2-4x-\cancel{1}}}$$

$$= \frac{\cancel{2}}{\cancel{2}\sqrt{-x^2-x}}$$

$y'(x)$ $\quad y = f(x) = \arcsin(2x+1)$

$$x = g(y) = \frac{\sin y - 1}{2} = \frac{\sin y}{2} - \frac{1}{2}$$

$$y' = \boxed{f'(x) = \frac{1}{D(f^{-1}(y))}} = \frac{1}{g'(y)} =$$

$$g'(y) = \frac{\cos y}{2}$$

$$\sin y = \sin(\arcsin(2x+1)) = 2x+1$$

$$\frac{\sin y - 1}{2}$$

$$\frac{2}{\cos y} = \frac{2}{\sqrt{1 - \sin^2 y}} =$$

$$= \frac{2}{\sqrt{1 - (2x+1)^2}} = \dots = \frac{1}{\sqrt{-x^2 - x}}$$