L.19 Generaliserede integraler

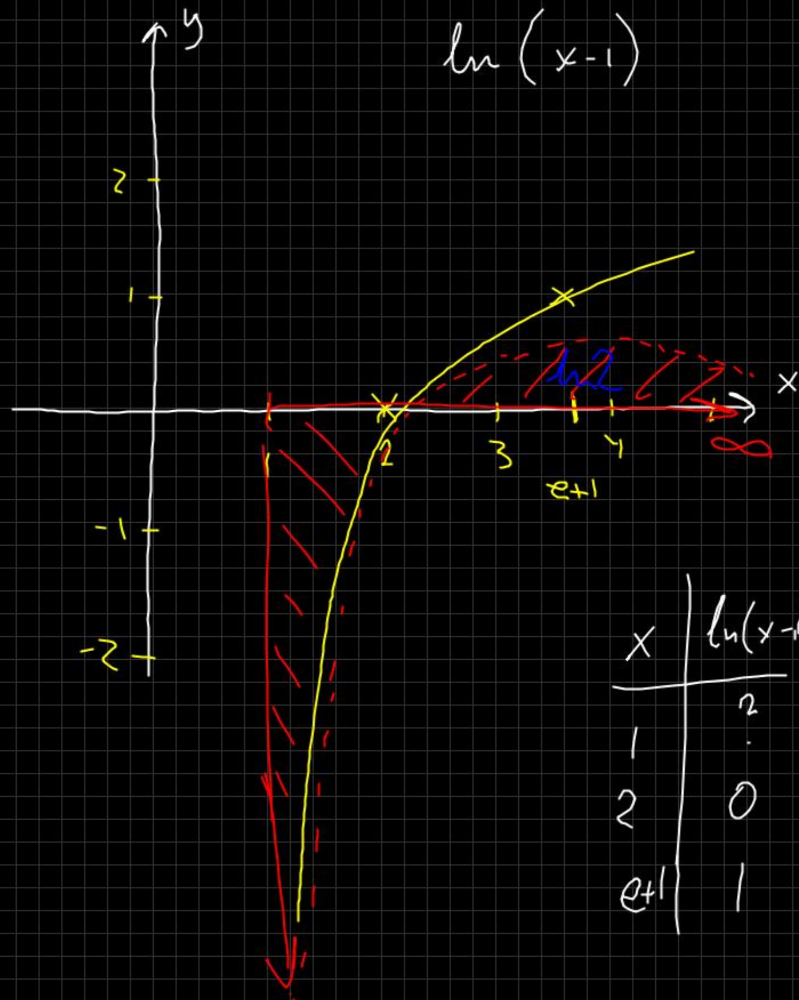
Ex. 1) $\displaystyle\int_1^{\infty} \frac{\ln(x-1)}{x^2}\,dx$

$\ln x$

e^x

$$= \int_1^2 \frac{\ln(x-1)}{x^2}\,dx + \boxed{\int_2^{\infty} \frac{\ln(x-1)}{x^2}\,dx}$$

$\ln(x-1)$



x	$\ln(x-1)$
1	?
2	0
e+1	1

$$\int_2^\infty \frac{\ln(x-1)}{x^2}\,dx = \int_2^\infty \overset{g}{\ln(x-1)} \cdot \overset{f}{x^{-2}}\,dx = F\cdot g - \int F g'\,dx =$$

$$= \left[-x^{-1}\cdot\ln(x-1)\right]_2^\infty + \boxed{\int_2^\infty \frac{1}{x}\cdot\frac{1}{x-1}\,dx} =$$

$$\cancel{+\ln x\frac{1}{x-1}+}$$

$$\cancel{\int_2^\infty \ln x\frac{1}{(x-1)^2}\,dx} =$$

$$A(x-1)+Bx=1$$

$$\left.\begin{aligned} A+B&=0\\ -A+0&=1\end{aligned}\right\}\begin{aligned}A&=-1\\ B&=1\end{aligned}$$

$$\boxed{\int_2^\infty\left(\frac{A}{x}+\frac{B}{x-1}\right)dx} = \left[-\frac{\ln(x-1)}{x}\right]_2^\infty + \int_2^\infty\left(\frac{-1}{x}+\frac{1}{x-1}\right)dx$$

Kom ihåg gränserna?

$$2 = \left[-\frac{\ln(x-1)}{x}\right]_2^{\infty} + \int_2^{\infty}\left(\frac{-1}{x} + \frac{1}{x-1}\right)dx = -\frac{\ln(x-1)}{x} - \ln x + \ln(x-1) + C$$

Döp om övre gräns till B

och beräkna $\lim_{B\to\infty}\int_2^B$

$$\ln\left(\frac{B-1}{B}\right)$$

$$\Rightarrow = \lim_{B\to\infty}\left[-\frac{\ln(B-1)}{B} - \left(-\frac{\overset{0}{\ln(2-1)}}{2}\right)\right] + \lim_{B\to\infty}\left[\underbrace{\left(-\ln B + \ln(B-1)\right)} - \left(-\ln 2 + \overset{0}{\ln(2-1)}\right)\right]$$

$$\overset{0}{\lim_{B\to\infty} -\frac{\ln(B-1)}{B}} + \overset{0}{\lim_{B\to\infty}\ln\left(\overset{1}{\frac{B-1}{B}}\right)} + \ln 2$$

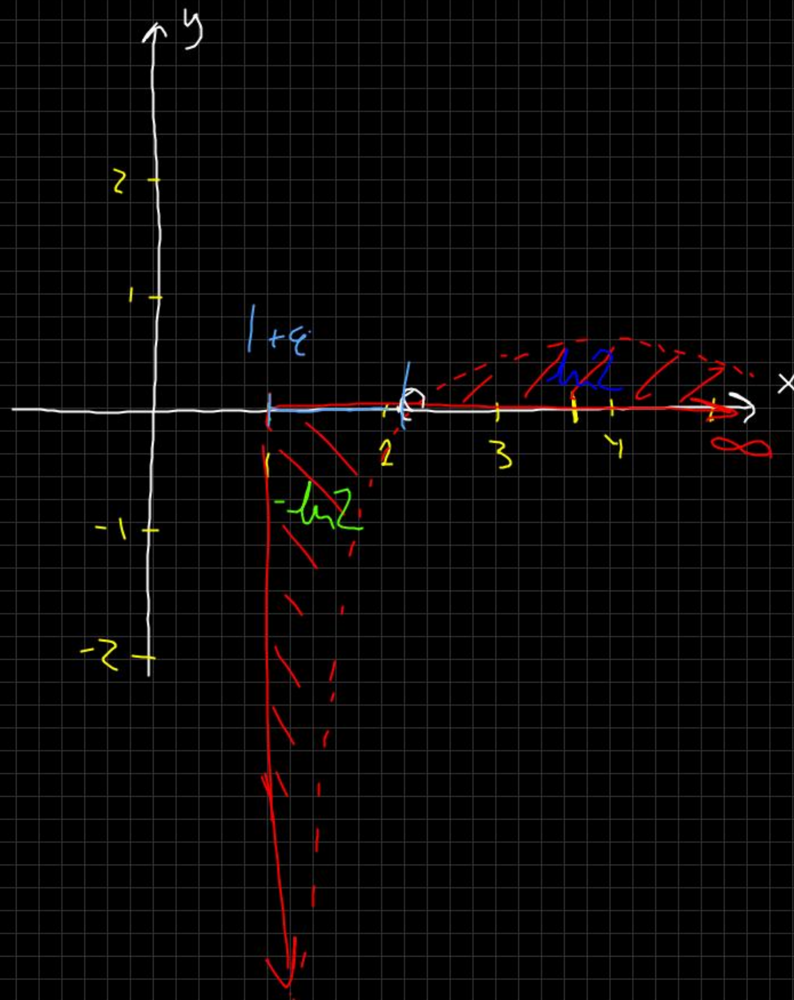
h. 19 x forts...

$$\int_1^\infty \frac{\ln(x-1)}{x^2}\,dx = \bigcirc$$

$$= \underbrace{\int_1^2 \frac{\ln(x-1)}{x^2}\,dx}_{=-\ln 2} + \underbrace{\int_2^\infty \frac{\ln(x-1)}{x^2}\,dx}_{\ln 2}$$

problem

$$\left[-\frac{\ln(x-1)}{x} - \ln x + \ln(x-1) + C\right]$$



1) döp om nedre gräns till $1+\varepsilon$.

2) låt $\varepsilon \to 0$.

$$\lim_{\varepsilon \to 0} \left[-\frac{\ln(x-1)}{x} - \ln x + \ln(x-1) \right]_{1+\varepsilon}^{2} =$$

$$\lim_{\varepsilon \to 0} \left(\left(-\underbrace{\frac{\ln(2-1)}{2}}_{=0} - \ln 2 + \underbrace{\ln(2-1)}_{=0} \right) - \left(-\frac{\ln(1+\varepsilon-1)}{1+\varepsilon} - \ln(1+\varepsilon) + \ln(1+\varepsilon-1) \right) \right)$$

$$\lim_{\varepsilon \to 0} \left(-\ln 2 + \frac{\ln \varepsilon}{1+\varepsilon} + \ln(1+\varepsilon) - \ln \varepsilon \right)$$

$$\lim_{\varepsilon \to 0} \left(\boxed{\ln(1+\varepsilon)} + \boxed{\left(\frac{1}{1+\varepsilon} - 1 \right) \ln \varepsilon} - \ln 2 \right) = \underline{\underline{-\ln 2}}$$

$$\boxed{\frac{1}{1+\varepsilon} - \frac{1+\varepsilon}{1+\varepsilon}} = \frac{1-(1+\varepsilon)}{1+\varepsilon} = -\frac{\varepsilon}{1+\varepsilon} \to 0 \text{ om } \varepsilon \to 0$$

$$\boxed{\ln(1+\varepsilon)} \to 0 \text{ om } \varepsilon \to 0$$