L.19 Generaliserede integraler

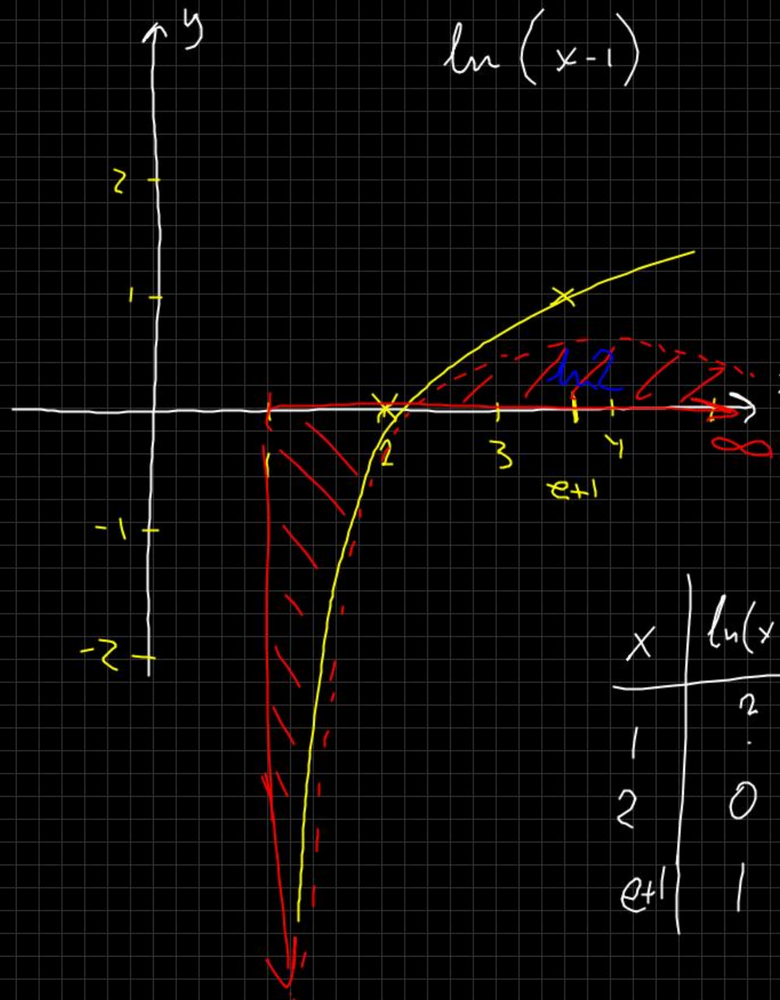
Ex. 1) $$\int_1^{\infty} \frac{\ln(x-1)}{x^2}\,dx$$

$\ln x$

e^x

$$= \int_1^2 \frac{\ln(x-1)}{x^2}\,dx + \boxed{\int_2^{\infty} \frac{\ln(x-1)}{x^2}\,dx}$$

$\ln(x-1)$



x	$\ln(x-1)$
1	?
2	0
e+1	1

$$\int_2^\infty \frac{\ln(x-1)}{x^2}\,dx = \int_2^\infty \overset{g}{\ln(x-1)} \cdot \overset{f}{x^{-2}}\,dx = F\cdot g - \int F g'\,dx =$$

$$= \left[-x^{-1}\cdot\ln(x-1)\right]_2^\infty + \boxed{\int_2^\infty \frac{1}{x}\cdot\frac{1}{x-1}\,dx} =$$

$$\cancel{+\ln x \frac{1}{x-1} +}$$

$$\cancel{\int_2^\infty \ln x \frac{1}{(x-1)^2}\,dx} =$$

$$A(x-1) + Bx = 1$$

$$\left.\begin{aligned} A+B&=0 \\ -A+0&=1 \end{aligned}\right\} \begin{aligned} A&=-1 \\ B&=1 \end{aligned}$$

$$\boxed{\int_2^\infty \left(\frac{A}{x} + \frac{B}{x-1}\right)dx} = \left[-\frac{\ln(x-1)}{x}\right]_2^\infty + \int_2^\infty \left(\frac{-1}{x} + \frac{1}{x-1}\right)dx$$

Kom ihåg gränserna?

$$2 = \left[-\frac{\ln(x-1)}{x}\right]_2^{\infty} + \int_2^{\infty}\left(\frac{-1}{x} + \frac{1}{x-1}\right)dx =$$

Döp om övre gräns till B och beräkna $\lim_{B\to\infty}\int_2^B$

$$\Rightarrow = \lim_{B\to\infty}\left[-\frac{\ln(B-1)}{B} - \left(-\overset{0}{\overset{\shortparallel}{\frac{\ln(2-1)}{2}}}\right)\right] + \lim_{B\to\infty}\left[\overbrace{\left(-\ln B + \ln(B-1)\right)}^{\ln\left(\frac{B-1}{B}\right)} - \left(-\ln 2 + \overset{0}{\overset{\shortparallel}{\ln(2-1)}}\right)\right]$$

$$\overset{0}{\overset{\shortparallel}{\lim_{B\to\infty} -\frac{\ln(B-1)}{B}}} + \overset{0}{\overset{\shortparallel}{\lim_{B\to\infty}\ln\left(\overset{1}{\overset{\shortparallel}{\frac{B-1}{B}}}\right)}} + \ln 2$$