2,85

a) $\int \underset{f}{2x}\,\underset{g}{(x^2+5)^5}\,dx = \frac{(x^2+5)^6}{6} + C$

Partiell integration

$$\boxed{\int f g\,dx = F\cdot g - \int F\cdot g'\,dx}$$

b) $\int \underset{f}{2x}\,\underbrace{(x^3+1)^2}_{g}\,dx = x^2\cdot(x^3+1)^2 - \int x^2\cdot 2(x^3+1)\cdot 3x^2\,dx$

$$= x^2\cdot(x^3+1)^2 - \int x^2\cdot 2(x^3+1)\cdot 3x^2\,dx = x^2(x^3+1)^2 - 6\int \underbrace{x^4}_{f'}\underbrace{(x^3+1)}_{g}\,dx$$

$$= x^2(x^6+2x^3+1) - 6\left[\frac{x^5}{5}\cdot(x^3+1) - \int \frac{x^5}{5}\cdot 3x^2\,dx\right] =$$

$$= x^8+2x^5+x^2 - \frac{6}{5}x^8 - \frac{6}{5}x^5 + 6\int \frac{3}{5}x^7\,dx =$$

$$= -\frac{1}{5}x^8 + \frac{4}{5}x^5 + x^2 + \frac{18}{5}\cdot\frac{x^8}{8} = \frac{5}{20}x^8 + \frac{16}{20}x^5 + \frac{20}{20}x^2 = \frac{x^2(5x^6+16x^3+20)}{20}$$

2.85

d)

$$\int \underbrace{2x}_{g} \cdot \underbrace{\cos x}_{f} \, dx = \left[F \cdot g - \int F \cdot g' \, dx \right] =$$

$$= \sin x \cdot 2x - \int \sin x \cdot 2 \, dx = 2x \sin x + 2 \cos x$$

$$= 2\left(x \sin x + \cos x \right) + C$$

$$y = 2x\sin x + 2\cos x + C$$

$$y' = \cancel{2 \cdot \sin x} + 2x \cdot \cos x \cancel{- 2\sin x} = 2x\cos x$$