2.79 a) $\int \sin 3x\,dx = \left[\begin{array}{l} t = 3x \\ \frac{dx}{dt} = \frac{1}{3} \Rightarrow dx = \frac{dt}{3} \end{array},\ x = g(t) = \frac{t}{3}\right] = \int \sin t \frac{dt}{3} =$

$= \frac{1}{3}\int \sin t\,dt = -\frac{\cos t}{3} + C = \boxed{-\frac{\cos 3x}{3} + C}$

c) $\int \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = \left[\begin{array}{l} t = \sqrt{x} \\ \frac{dx}{dt} = 2t \Rightarrow dx = 2t\,dt \end{array},\ x = g(t) = t^2,\ t \geq 0\right] = \int \frac{1+t}{t} \cdot 2t\,dt$

$= 2\int (1+t)\,dt = 2\left(t + \frac{t^2}{2}\right) + C = 2\left(\sqrt{x} + \frac{\sqrt{x}^2}{2}\right) + C = \boxed{2\sqrt{x} + x + C}$

$\int \left(\frac{1}{\sqrt{x}} + 1\right) dx = \int \left(x^{-\frac{1}{2}} + 1\right) dx = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + x + C = \boxed{2\sqrt{x} + x + C}$

2.85 a)

$$\int \underbrace{2x}_{g(x)} \underbrace{\left(x^2+5\right)^5}_{f(x)}\,dx = \left[\begin{array}{l} t = x^2+5 \qquad x = g(t) = (t-5)^{1/2} \\ \frac{dx}{dt} = \frac{1}{2}\cdot(t-5)^{-1/2} \qquad dx = \frac{dt}{2(t-5)^{1/2}} \end{array}\right] =$$

$$= \int 2(t-5)^{1/2}\cdot t^5 \cdot \frac{1}{2(t-5)^{1/2}}\,dt = \int t^5\,dt = \frac{t^6}{6}+C = \frac{(x^2+5)^6}{6}+C$$

F vera primitiv till f

$$\boxed{\int f(x)g(x)\,dx = F(x)\cdot g(x) - \int F(x)\cdot g'(x)\,dx}$$

$$= \frac{(x^2+5)^6}{6}\cdot 2x - \int \frac{(x^2+5)^6}{6\cdot 2x}\cdot 2\,dx =$$

$$= \frac{(x^2+5)^6}{6} - \int \underbrace{x^{-1}}_{f}\cdot\underbrace{(x^2+5)^6}_{g}\,dx = \frac{(x^2+5)^6}{6} - \frac{1}{6}\int \ln x\cdot(x^2+5)^6 - \int \ln x\cdot 6\cdot 2x(x^2+5)^5\,dx =$$

2.85 b)

$$\int \underbrace{2x}_{g}\,\overbrace{(x^3+1)^2}^{f}\,dx = \frac{(x^3+1)^3}{3\cdot 3x^2}\cdot 2x - \int \frac{(x^3+1)^3}{3\cdot 3x^2}\cdot 2\,dx$$

(f and g are also written above, over $2x$ and $(x^3+1)^2$.)

aja baja! (written under the first fraction, which is crossed out)

ajabaja (written over the crossed-out fraction in the second integral)

? (written under the second integral)

Testa tvärtom?

$$\int 2x(x^3+1)^2\,dx = x^2(x^3+1)^2 - \int x^2\cdot 2(x^3+1)\cdot 3x^2\,dx$$

$$= x^2(x^3+1)^2 - \int 6x^4(x^3+1)\,dx = x^2(x^3+1)^2 - \int (6x^7+6x^4)\,dx =$$

$$x^2(x^6+2x^3+1) - \frac{6x^8}{8} - \frac{6x^5}{5} + C = x^8+2x^5+x^2-\frac{3}{4}x^8-\frac{6}{5}x^5+C = \boxed{\frac{1}{4}x^8+\frac{4}{5}x^5+x^2+C}$$

(2.95) a) $\int \frac{4x^3-3x^2-2}{x^2-x}\,dx = \int \left(4x+1+\boxed{\frac{x-2}{x^2-x}}\right)dx =$

$$4x+1$$
$$4x^3-3x^2+0x-2 \;\big|\; x^2-x$$
$$-(4x^3-4x^2)$$
$$0+x^2+0x-2$$
$$-(x^2-x)$$
$$0+x-2$$

$$\boxed{\frac{x-2}{x(x-1)}} = \frac{A}{x}+\frac{B}{x-1} = \boxed{\frac{2}{x}-\frac{1}{x-1}}$$

$$= \frac{A(x-1)+B\cdot x}{x(x-1)} = \frac{x-2}{x(x-1)}$$

x-term: $A+B=1$

konstant: $-A=-2$

$A=2 \Rightarrow B=-1$

$$\int\left(4x+1+\frac{2}{x}-\frac{1}{x-1}\right)dx = 2x^2+x+2\ln x-\ln(x-1)+C.$$