

del 2

L. 17

Partiellbräksuppdelning :

Ex.

$$\int \frac{x^4 + 3x^3 + x + 4}{x^3 + 3x^2 - 4} dx$$

$$\begin{array}{r} x \\ \hline x^4 + 3x^3 + 0x^2 + x + 4 \quad | \quad x^3 + 3x^2 - 4 \\ - (x^4 + 3x^3 \quad - 4x) \\ \hline 0 + 0 + 0 + 5x + 4 \end{array}$$

$$= \int \left( x + \frac{5x+4}{x^3+3x^2-4} \right) dx$$

Rationella uttryck

$$\boxed{\frac{f(x)}{g(x)}}, \quad \begin{array}{l} f(x), g(x) \\ \text{polynom} \end{array}$$

I) grad  $f(x) \geq$  grad  $g(x)$  $\Rightarrow$  polynomdivision

II) Faktorisera nämnaren så långt som möjligt.

III) Partiellbräksuppdelning

IV) Integrera

Fokus på andra termen i  $\int$  Partialbräksuppdelning.

$$\frac{5x+4}{x^3+3x^2-4} = \left[ \begin{array}{l} \text{om } x=1 \\ x^3+3x^2-4 = 1+3-4=0 \Rightarrow (x-1) \text{ är en faktor!} \end{array} \right] =$$

$$= \frac{5x+4}{(x-1)(x^2+4x+4)} = \frac{A}{(x-1)} + \frac{\cancel{Bx+C}}{\cancel{(x^2+4x+4)}} = \frac{A}{(x-1)} + \frac{B}{(x+2)^2}$$

$$= \frac{5x+4}{(x-1)(x+2)^2} = \frac{A}{(x-1)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$\begin{array}{r} x^2+4x+4 \\ \hline x^3+3x^2+0x-4 \quad \boxed{x-1} \\ - (x^3-x^2) \\ \hline 0+4x^2+0x-4 \\ - (4x^2-4x) \\ \hline 0+4x-4 \\ - (4x-4) \\ \hline 0 \quad 0 \end{array}$$

$$\frac{A=1}{x-1} + \frac{B=-1}{x+2} + \frac{C=2}{(x+2)^2} = \frac{A(x+2)^2 + B(x-1)(x+2) + C(x-1)}{(x-1)(x+2)^2} = \frac{5x+4}{(x-1)(x+2)^2}$$

$$\begin{array}{l} x^2\text{-terms} \\ x\text{-terms} \\ \text{konstanten} \end{array} \left\{ \begin{array}{l} A + B + 0 = 0 \\ 4A + B + C = 5 \\ 4A - 2B - C = 4 \end{array} \right. = \begin{array}{l} -4 \\ -4 \\ -4 \end{array} \left\{ \begin{array}{l} A + B + 0 = 0 \\ 0 - 3B + C = 5 \\ 0 - 6B - C = 4 \end{array} \right. = \begin{array}{l} 0 \\ -2 \\ -2 \end{array}$$

$$= \left\{ \begin{array}{l} A + B + 0 = 0 \\ 0 - 3B + C = 5 \\ 0 + 0 - 3C = -6 \end{array} \right. \Rightarrow \begin{array}{l} A - 1 = 0 \Rightarrow A = 1 \\ -3B + 2 = 5 \Rightarrow B = -1 \\ C = 2 \end{array}$$

$$\frac{1}{x-1} - \frac{1}{(x+2)} + \frac{2}{(x+2)^2}$$

$$2(x+2)^{-2}$$
$$\frac{2(x+2)^{-1}}{-1}$$

$$\Rightarrow \int \frac{x^4 + 3x^3 + x + 4}{x^3 + 3x^2 - 4} dx = \int \left( x + \frac{1}{x-1} - \frac{1}{x+2} + \frac{2}{(x+2)^2} \right) dx =$$

$$= \frac{x^2}{2} + \ln|x-1| - \ln|x+2| - \frac{2}{x+2} + C$$