Variabelsubstitution Lektion 14

forts.

$$\int \frac{1}{x+\sqrt{x}}\,dx = \quad t=\sqrt{x}\;,\; x=t^2 \quad = \int \frac{1}{t^2+t}\,dx$$

?

Kedjeregel: Låt F vara primitiv funktion till f.

$$\big(F(g(t))\big)' = F'(g(t))\cdot g'(t) = f(g(t))\cdot g'(t)$$

Alltså

$$\int \left(F(g(t))\right)' \, dt = \int f(g(t)) \cdot g'(t) \, dt$$

$$F(g(t)) + C = \int f(g(t)) \, g'(t) \, dt$$

Antag att jag vill bestämma $\int f(x)\,dx$ med bytet att $x = g(t)$

svårt

$x = g(t)$

$$\int f(x)\,dx = \underline{F(x) + C} = F(g(t)) + C = \int f(g(t)) \cdot g'(t)\,dt$$

1 4 3 2

Minnesregel:

$$\int f(x)\,dx = \boxed{F(x)+C} = F(g(t))+C = \int f(g(t))\cdot g'(t)\,dt$$

1 4 3 2

Ex. 1)

$$\int \frac{1}{x+\sqrt{x}}\,dx = \boxed{\begin{array}{l} t=\sqrt{x}\ ,\ x=g(t)=t^2\ ,\ t>0 \\ \frac{dx}{dt}=g'(t)=2t \Rightarrow dx = 2t\,dt \end{array}} = \int \frac{1}{t^2+t}\,2t\,dt =$$

$$= \int \frac{1}{\cancel{t}(t+1)}\cdot 2\cancel{t}\,dt = \int \frac{2}{t+1}\,dt = 2\int \frac{1}{t+1}\,dt = 2\ln(t+1)+C$$

$$= 2\ln(\sqrt{x}+1)+C$$

<u>Minnesregel:</u> $\int f(x)\,dx = F(x) + C = F(g(t)) + C = \int f(g(t)) \cdot g'(t)\,dt$

Ex 2.)

$$\int \frac{1}{x \ln x}\,dx = \left[\begin{array}{ll} t = \ln x & x = g(t) = e^t \\ \frac{dx}{dt} = e^t & \Rightarrow dx = e^t\,dt \end{array}\right] = \int \frac{1}{\cancel{e^t} \cdot t} \cdot \cancel{e^t}\,dt =$$

$$= \int \frac{1}{t}\,dt = \ln t + C = \ln(\ln x) + C$$

Minnesregel:

$$\int f(x)\,dx = F(x)+C = F(g(t))+C = \int f(g(t))\cdot g'(t)\,dt$$

Ex. 3)

$$\int x\sqrt{x+2}\,dx = \left[\begin{array}{ll} t=\sqrt{x+2} & x=g(t)=t^2-2 \\ \dfrac{dx}{dt}=2t & \Rightarrow\ dx=2t\,dt \end{array}\right] = \int (t^2-2)\cdot t\cdot 2t\,dt =$$

$$= \int (2t^4-4t^2)\,dt = \frac{2t^5}{5} - \frac{4t^3}{3} + C = \frac{2(x+2)^{5/2}}{5} - \frac{4(x+2)^{3/2}}{3} + C$$

Minnesregel: $\int f(x)\,dx = F(x)+C = F(g(t))+C = \int f(g(t))\cdot g'(t)\,dt$

Ex. 4) $\boxed{\int \frac{1}{x^2+1}\,dx} = \left[\begin{array}{l} t = x^2, \quad x = g(t) = t^{1/2} \\ \frac{dx}{dt} = \frac{1}{2}\cdot t^{-1/2} \Rightarrow dx = \frac{1}{2}t^{-1/2}\,dt \end{array}\right] = \int \frac{1}{t+1}\cdot\frac{1}{2}t^{-1/2}\,dt =$

$= \left[\begin{array}{l} u = t+1, \quad t = u-1 = g(u) \\ \frac{dt}{du} = 1 \Rightarrow dt = du \end{array}\right] = \int \frac{1}{u\cancel{-1}+\cancel{1}}\cdot\frac{1}{2}(u-1)^{-1/2}\,du =$

$t = x^2+1$

<u>Minnesregel:</u> $\int f(x)\,dx = F(x)+C = F(g(t))+C = \int f(g(t))\cdot g'(t)\,dt$

$$\int \frac{1}{x^2+1}\,dx = \left[\begin{array}{l} t = x^2+1 \quad , \quad x = g(t) = (t-1)^{1/2} \\ \frac{dx}{dt} = \frac{1}{2}\cdot(t-1)^{-1/2} \Rightarrow dx = \frac{1}{2}(t-1)^{-1/2}\,dt \end{array}\right] = \int \frac{1}{t}\cdot\frac{1}{2}(t-1)^{-1/2}\,dt$$

<u>Idé:</u> Känner igen denna funktion: $f(x) = \dfrac{1}{x^2+1}$ som derivatan

på $\arctan x = F(x)$