

Variabelsubstitution

Lektion 14

forts.

$$\int \frac{1}{x + \sqrt{x}} dx = \int \frac{1}{t^2 + t} dx$$

$$t = \sqrt{x}, \quad x = t^2$$



Kedjeregeln: Låt F vara primitiv funktion till f .

$$(F(g(t)))' = F'(g(t)) \cdot g'(t) = f(g(t)) \cdot g'(t)$$

Alltså

$$\int (F(g(t)))' dt = \int f(g(t)) \cdot g'(t) dt$$
$$F(g(t)) + C = \int f(g(t)) g'(t) dt$$

Antag att jag vill bestämma $\int f(x) dx$ med bytet att $x = g(t)$

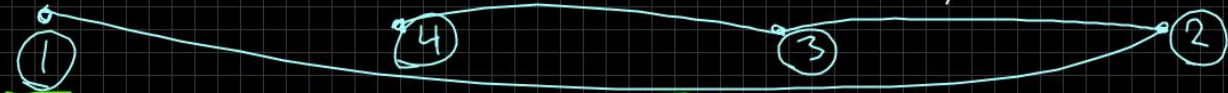
svart \rightarrow

$$\int f(x) dx = \underbrace{F(x) + C}_{(4)} \stackrel{x=g(t)}{=} F(g(t)) + C = \int f(g(t)) g'(t) dt$$

(1) (2) (3)

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Minnesregel: $\int f(x) dx = \boxed{F(x) + C} = F(g(t)) + C = \int f(g(t) \cdot g'(t)) dt$



Ex. 1) $\int \frac{1}{x+\sqrt{x}} dx = \left[\begin{array}{l} t = \sqrt{x}, \quad x = g(t) = t^2, \quad t > 0 \\ \frac{dx}{dt} = g'(t) = 2t \Rightarrow dx = 2t dt \end{array} \right] = \int \frac{1}{t^2 + t} 2t dt =$

$= \int \frac{1}{\cancel{t}(t+1)} \cdot \cancel{2} t dt = \int \frac{2}{t+1} dt = 2 \int \frac{1}{t+1} dt = 2 \ln|t+1| + C$

$= 2 \ln(\sqrt{x}+1) + C$

Minnesregel:

$$\int f(x) dx = F(x) + C = F(g(t)) + C = \int f(g(t) \cdot g'(t)) dt$$

Ex 2.)

$$\int \frac{1}{x \ln x} dx = \left[\begin{array}{l} t = \ln x \quad x = g(t) = e^t \\ \frac{dx}{dt} = e^t \Rightarrow dx = e^t dt \end{array} \right] = \int \frac{1}{\cancel{e^t} \cdot t} \cdot \cancel{e^t} dt =$$

$$= \int \frac{1}{t} dt = \ln t + C = \ln(\ln x) + C$$

Minnesregel:

$$\int f(x) dx = F(x) + C = F(g(t)) + C = \int f(g(t) \cdot g'(t)) dt$$

Ex. 3)

$$\int x \sqrt{x+2} dx = \left[\begin{array}{l} t = \sqrt{x+2} \quad x = g(t) = t^2 - 2 \\ \frac{dx}{dt} = 2t \Rightarrow dx = 2t dt \end{array} \right] = \int (t^2 - 2) \cdot t \cdot 2t dt =$$

$$= \int (2t^4 - 4t^2) dt = \frac{2t^5}{5} - \frac{4t^3}{3} + C$$

$$= \frac{2(x+2)^{5/2}}{5} - \frac{4(x+2)^{3/2}}{3} + C$$

Minnesregel: $\int f(x) dx = F(x) + C = F(g(t)) + C = \int f(g(t) \cdot g'(t)) dt$

Ex. 4) $\int \frac{1}{x^2 + 1} dx = \left[\begin{array}{l} t = x^2, \quad x = g(t) = t^{1/2} \\ \frac{dx}{dt} = \frac{1}{2} t^{-1/2} \Rightarrow dx = \frac{1}{2} t^{-1/2} dt \end{array} \right] = \int \frac{1}{t+1} \cdot \frac{1}{2} t^{-1/2} dt =$

$= \left[\begin{array}{l} u = t+1, \quad t = u-1 = g(u) \\ \frac{dt}{du} = 1 \Rightarrow dt = du \end{array} \right] = \int \frac{1}{\cancel{u-1} + \cancel{1}} \cdot \frac{1}{2} (u-1)^{-1/2} du =$

$t = x^2 + 1$

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Minnesregel: $\int f(x) dx = F(x) + C = F(g(t)) + C = \int f(g(t)) \cdot g'(t) dt$

$$\int \frac{1}{x^2+1} dx = \left[\begin{array}{l} t = x^2+1, \quad x = g(t) = (t-1)^{1/2} \\ \frac{dx}{dt} = \frac{1}{2} \cdot (t-1)^{-1/2} \Rightarrow dx = \frac{1}{2} (t-1)^{-1/2} dt \end{array} \right] = \int \frac{1}{t} \cdot \frac{1}{2} (t-1)^{-1/2} dt$$

Ide: Känner igen denna funktion: $f(x) = \frac{1}{x^2+1}$ som derivatan
på $\arctan x = \bar{F}(x)$

