L13

2.9 Beräkna

a)
$$\int_0^2 9x^2\,dx = \Big[\,3x^3\,\Big]_0^2 = (3\cdot 2^3) - (3\cdot 0^3) = 24.$$

b)
$$\int_0^\pi \sin\left(2x - \frac{\pi}{4}\right)dx = \left[-\frac{\cos\left(2x-\frac{\pi}{4}\right)}{2}\right]_0^\pi = \left(-\frac{\cos\left(2\pi-\frac{\pi}{4}\right)}{2}\right) - \left(-\frac{\cos\left(0-\frac{\pi}{4}\right)}{2}\right) = -\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = 0$$

c)
$$\int_0^1 (1-\sqrt{x})\,dx = \left[x - \frac{2\cdot x^{3/2}}{3}\right]_0^1 = \left(1 - \frac{2}{3}\right) - (0) = \frac{1}{3}$$

2.9 d) $\int_0^1 (2x-1)\,dx + \int_1^2 (2x-1)\,dx = \int_0^2 (2x-1)\,dx = \left[x^2 - x\right]_0^2 =$

$= (2^2 - 2) - (0^2 - 0) = 2$

$$\boxed{\cancel{D}\left(\boxed{f(g(x))}\right) = \int f'(g(x)) \cdot g'(x)}$$

$$\int \underline{f'(g(x)) \cdot g'(x)\,dx}$$

Variabel subst.

$$\int \underbrace{(2x^2+1)^7}_{f'} \cdot \underbrace{4x}_{g'}\, dx = \left[\begin{array}{l} t = 2x^2+1 \\ \dfrac{dt}{dx} = 4x \quad \Rightarrow \quad dt = 4x\,dx \end{array}\right] =$$

$$\int (t)^7 \cdot dt = \frac{t^8}{8} + C = \frac{(2x^2+1)^8}{8} + C$$

Ex - Variabelsubstitution

$$\int \frac{1}{x+\sqrt{x}}\,dx = \left[\begin{array}{l} t = \sqrt{x} = x^{1/2} \Rightarrow x = t^2 = g(t), \\ \frac{dt}{dx} = \frac{1}{2} \cdot x^{-1/2} \Rightarrow dt = \frac{1}{2\sqrt{x}}\,dx \\ \qquad \Rightarrow g'(t) = 2t \end{array}\right]$$

$\Rightarrow$ fortsättning nästa lektion!

Måndag $\Rightarrow$

	Primitiva funktioner	
12	2.1, 2.2, 2.3, 2.6, 2.7	*2.4, 2.5, 2.8*
	Allmänna egenskaper hos integraler	
13	2.9, 2.10, 2.11, 2.13, 2.15, 2.20	*2,12, 2.14, 2.16, 2.18, 2.19*