# Primitiva funktioner    Lektion 12.

Matematikkurser för skolan    ?

$\Downarrow$

Repetition    Ma3c

Bestäm en primitiv funktion $F(x)$ till funktionerna

**2.1**

a) $f(x) = 2x$

$F(x) = x^2$

b) $f(x) = 7$

$F(x) = 7x$

c) $f(x) = 3x^7 + 4x$

$F(x) = \frac{3x^8}{8} + 2x^2$

d) $f(x) = \frac{x}{6} + \frac{6}{x}$ , $x > 0$

$F(x) = \frac{x^2}{12} + 6\ln x$

e) $f(x) = 7e^x + e$

$F(x) = 7e^x + ex$

f) $f(x) = 3\sin x - 4\cos x$

$F(x) = -3\cos x - 4\sin x$

g) $f(x) = x\sqrt{x} = x^{3/2}$

$F(x) = \frac{2}{5}x^{5/2} = \frac{2x^2\sqrt{x}}{5}$

h) $f(x) = \frac{x^3}{4} + \frac{4}{x^3}$

$F(x) = \frac{x^4}{16} - \frac{2}{x^2}$

**2.2**

a) $f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$

$F(x) = 2x^{1/2} = 2\sqrt{x}$

b) $f(x) = \cos 8x - 8\cos x$

$F(x) = \frac{\sin 8x}{8} - 8\sin x$

c) $f(x) = \frac{3x^2 - 2}{x^2} = 3 - \frac{2}{x^2} = 3 - 2x^{-2}$

$F(x) = 3x + 2x^{-1} = 3x + \frac{2}{x}$

d) $f(x) = (5x+3)^7$

$F(x) = \frac{(5x+3)^8}{8 \cdot 5} = \frac{(5x+3)^8}{40}$

e) $f(x) = e^{5x+2}$

$F(x) = \frac{e^{5x+2}}{5}$

f) $f(x) = 6\cos\frac{2\pi}{3} - 2\sin x\cos x$ 2

$F(x) = \frac{3 \cdot \cancel{6}}{2\pi}\sin\frac{2\cancel{x}}{3} + \cos^2 x = 9\sin\frac{2x}{3} + \cos^2 x$

g) $f(x) = \frac{1}{4x+1}$

$F(x) = \frac{1}{4}\ln(4x+1)$

h) $f(x) = (x^2+1)^2 = x^4 + 2x^2 + 1$

$\left(F(x) = \frac{(x^2+1)^3}{3 \cdot 2x} = \frac{(x^2+1)^3}{6x}\right) = \frac{x^5}{5} + \frac{2x^3}{3} + x$

2.3 Bestäm de primitiva funktionerna $F(x)$ som uppfyller nedanstående villkor

8.42 Kopplingsschema löst här

a) $f(x) = 6x^2 + 1$ och $F(1) = 0$

$F(x) = 2x^3 + x + C$

$$\begin{cases} F(1) = 2 \cdot 1^3 + 1 + C = 2 + 1 + C = 3 + C \\ F(1) = 0 \end{cases}$$

$3 + C = 0 \quad C = -3$ $\quad$ <u>dvs</u> $\quad \underline{\underline{F(x) = 2x^3 + x - 3}}$

b) $f(x) = x + \cos 2x$ och $F(\pi) = \frac{\pi^2}{4}$

$F(x) = \frac{x^2}{2} + \frac{1}{2}\sin 2x + C$

$$\begin{cases} F(\pi) = \frac{\pi^2}{2} + \frac{1}{2}\underbrace{\sin 2 \cdot \pi}_{0} + C = \frac{\pi^2}{2} + C \\ F(\pi) = \frac{\pi^2}{4} \end{cases}$$

$\frac{\pi^2}{2} + C = \frac{\pi^3}{4}$ , $C = -\frac{\pi^2}{4}$ $\quad \underline{\underline{F(x) = \frac{x^2}{2} + \frac{1}{2}\sin 2x - \frac{\pi^2}{4}}}$

2.6

(2.7) Bestäm en primitiv funktion $F(x)$ till funktionerna

a) $f_1(x) = 2\sin x\cos x \rightarrow f_2(x) = \sin 2x$ (formelsamling)

$F_1(x) = \sin^2 x + C$ ex. $F_1(x) = \sin^2 x - 1 \; (= -\cos^2 x)$

$F_2(x) = \frac{-\cos 2x}{2} + C$ ex. $F_2(x) = \frac{1}{2} - \frac{\cos 2x}{2} \; (+C)$

~~b) $f(x) = \cos^2 x - \sin^2 x = (\cos x + \sin x)(\cos x - \sin x) =$~~

~~$F(x) = \frac{\cos^3 x}{3\sin x} - \frac{\sin^3 x}{3\cos x}$~~

~~$= \sqrt{2}\sin(x + \pi/4) \cdot \sqrt{2}\sin(x - \pi/4) =$~~

~~$= 2(\sin x\cos \pi/4 - \sin \pi/4 \cos x)(\sin x \cos \pi/4 + \sin \pi/4 \cos x)$~~

~~$= 2\left(\frac{1}{\sqrt{2}}\sin x - \frac{1}{\sqrt{2}}\cos x\right)\left(\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x\right) =$~~

b) $f(x) = \cos^2 x - \sin^2 x = \cos 2x$ (formelsamling)

$F(x) = +\frac{\sin 2x}{2} \; (+C)$

c) $f(x) = \frac{e^{3x} - e^{x}}{e^{2x}} = e^{x} - e^{-x}$

$F(x) = e^{x} + e^{x} = 2e^{x} \; (+C)$

d) $f(x) = 8\sin 3x\cos 3x = 4\sin 6x$

$F(x) = -\frac{4\cos 6x}{6} = -\frac{2}{3}\cos 6x \; (+C)$

e) $f(x) = \sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$

$F(x) = \frac{1}{2}x - \frac{\sin 2x}{4} + C$

f) $y = f(x) = \frac{1}{1+x^2} \Rightarrow x$

$F(x) = \frac{\ln(1+x^2)}{2x} + C$

(2.)

$F(x) = \arctan x$

Lektion 13 — Repetition Ma3c

2.9
10
11
13
15

Allmänna egenskaper hos integraler

$$\int f(x)\,dx$$

**2.9**

a) $\int_0^2 9x^2\,dx = \left[3x^3\right]_0^2 = (3\cdot 2^3)-(3\cdot 0) = 3\cdot 8 = 24$

b) $\int_0^{\pi} \sin\left(2x-\frac{\pi}{4}\right)dx = \left[-\frac{\cos\left(2x-\frac{\pi}{4}\right)}{2}\right]_0^{\pi} = \left(-\frac{\cos\left(2\pi-\frac{\pi}{4}\right)}{2}\right)-\left(-\frac{\cos\left(0-\frac{\pi}{4}\right)}{2}\right)$

$= \left(-\frac{1}{2}\cdot\frac{1}{\sqrt2}\right)-\left(-\frac{1}{2}\cdot\frac{1}{\sqrt2}\right) = -\frac{1}{2\sqrt2}+\frac{1}{2\sqrt2} = 0$

c) $\int_0^1 (1-\sqrt{x})\,dx = \left[x-\frac{2x^{3/2}}{3}\right]_0^1 = \left(1-\frac{2}{3}\right)-(0) = \frac{1}{3}$

d) $\int_0^1 (2x-1)\,dx + \int_1^2 (2x-1)\,dx = \int_0^2 (2x-1)\,dx = \left[x^2-x\right]_0^2 = (2^2-2)-(0) = 2$

**2.10** Beräkna

a) $\int_0^3 (6x^2-6x+12)\,dx = \left[2x^3-3x^2+12x\right]_0^3 = (2\cdot 3^3-3\cdot 3^2+12\cdot 3)-(0) =$
$54-27+36 = 63$

b) $\int_{-2}^2 (x^4-x^2+2)\,dx = \left[\frac{x^5}{5}-\frac{x^3}{3}+2x\right]_{-2}^2 = \left(+\frac{2^5}{5}-\frac{2^3}{3}+2^2\right)-\left(-\frac{2^5}{5}+\frac{2^3}{3}-2^2\right) = 2\left(\frac{2^5}{5}-\frac{2^3}{3}+2^2\right)$

$= 2^3\left(\frac{2^3}{5}-\frac{2}{3}+1\right) = 8\left(\frac{8}{5}-\frac{2}{3}+1\right) = 8\left(\frac{24}{15}-\frac{10}{15}+\frac{15}{15}\right) = 8\cdot\frac{29}{15} = \frac{232}{15}$

**2.11**

$F = -k\cdot y$ = Fjäderkraften

$y$ = förlängning av gummiband = 0,36m − 0,30 = 0,06 m

$k$ = proportionalitetskonstant = 0,03 N/m

~~$F = 0{,}03\cdot 0{,}06 =$~~

Arbetet = $W = \int F\,dy = \cancel{\frac{ky^2}{}} = \int_0^{0{,}06} k\cdot y\,dy =$

$= \left[\frac{ky^2}{2}\right]_0^{0{,}06} = \left[\frac{0{,}03\cdot y^2}{2}\right]_0^{0{,}06} = \frac{0{,}03\cdot 0{,}06^2}{2} = 54\,\mu J$

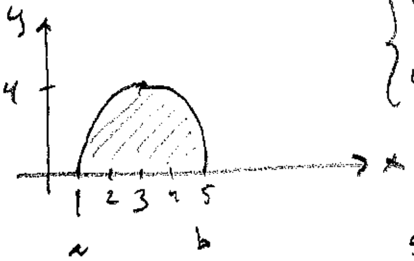
Svar: Arbetet blir $W = 54\,\mu J$

**2.13** Förenkla

a) $\int_a^b k \cdot dx$, $k$ = konstant $= k \cdot b - k \cdot a = \underline{\underline{k(b-a)}}$

b) $\int_a^b dx = \int_a^b 1\,dx = \left[\, x \,\right]_a^b = \underline{\underline{b-a}}$

**2.15** Beräkna arean mellan kurvan $y = 4-(x-3)^2$ och x-axeln.




$$\begin{cases} y = 4-(x-3)^2 \\ y = 0 \end{cases} \Rightarrow (x-3)^2 = 4$$

$$x+3 = \pm 2$$

$$\begin{cases} x_1 = 1 = a \\ x_2 = 5 = b \end{cases}$$

$$A = \int_1^5 4-(x-3)^2\,dx = \left[4x - \frac{(x-3)^3}{3}\right]_1^5 = \cancel{\left(4 - \frac{8}{3}\right) - (20}$$

$$= \left(20 - \frac{(2)^3}{3}\right) - \left(4 + \frac{(-2)^3}{3}\right) = \cancel{20 - 4 - 16} = 16 - \frac{16}{3} = \frac{48-16}{3}$$

$\underline{\text{Svar:}}$ $A = \frac{32}{3}$

Lektion 14

Variabelsubstitution vid primitiva funktioner

Ex $\int \frac{1}{x+\sqrt{x}}\,dx = \left[\ t=\sqrt{x} \Leftrightarrow x=t^2 \quad (t>0)\ \right] =$

$= \int \frac{1}{t^2+t}\,dx \quad ?$

<u>Kedjeregeln:</u> Låt $F$ vara primitiv funktion till $f$

$$\left(F(g(t))\right)' = F'(g(t))\cdot \underbrace{g'(t)}_{\text{inre derivata}} = f(g(t))\cdot g'(t)$$

$$\int \left(F(g(t))\right)'\,dt = \int f(g(t))\cdot g'(t)\,dt \Rightarrow$$

$$\boxed{F(g(t)) + C = \int f\ g(t)\cdot g'(t)\,dt}$$

Antag att vi vill bestämma $\int f(x)\,dx$ med bytet $x = g(t)$.

$$\underbrace{\int f(x)\,dx}_{\text{Det här vill jag bestämma}} = \underbrace{F(x)+C}_{\text{Det här svaret vill jag uppnå}} \overset{x=g(t)}{=} F(g(t))+C = \int f(g(t))\cdot g'(t)\,dt$$

$$1 = 4 \leftarrow 3 \leftarrow 2$$

Ex: $$\int \frac{1}{x+\sqrt{x}}\,dx \overset{x=g(t)}{=} \left[\begin{array}{l} t=\sqrt{x} \Rightarrow x=g(t)=t^2,\ t>0 \\ g'(t)=2t \end{array}\right] =$$

$$= \int \frac{1}{t^2+t}\cdot 2t\,dt = \int \frac{2}{t+1}\,dt = 2\ln|t+1| + C$$

Men vi ställde frågan i $x$!

$$= 2\ln(\sqrt{x}+1) + C$$

Mål

$$\boxed{\int \frac{p(x)}{q(x)}\,dx}$$

2.20

a) $\int (x^2+1)^3\, 2x\, dx = \left[\begin{array}{l} t = x^2+1 \Rightarrow x = g(t) = \sqrt{t-1}, \quad t>1 \\ \frac{dx}{dt} = g'(t) = \frac{1}{2}\frac{1}{\sqrt{t-1}} \end{array}\right]$

$\int t^3 \cdot \cancel{2}\cancel{\sqrt{t-1}} \cdot \frac{1}{\cancel{2}} \cdot \frac{1}{\cancel{\sqrt{t-1}}}\, dt = \int t^3\, dt = \cancel{\frac{t^4}{4} + C}$

$= \frac{(x^2+1)^4}{4} + C$

$\boxed{\begin{array}{l} \underline{\text{Minnesregel:}} \quad \frac{dx}{dt} = g'(t) \Rightarrow dx = g'(t)dt \\ f(x)\,dx = f(g(t))\cdot g'(t)dt \end{array}}$

b) $\int (x^3+2)^9 \cdot x^2\, dx =$

$= \left[\begin{array}{l} t = x^3+2 \Rightarrow x = g(t) = (t-2)^{1/3} \\ \frac{dx}{dt} = \frac{1}{3}\cdot(t-2)^{-2/3} \end{array}\right]$

$= \int t^9 \cdot \frac{1\cdot(t-2)^{2/3}}{3\cdot\cancel{(t-2)^{2/3}}} \cdot dt = \int t^9 \cdot \frac{1}{3}\, dt = \cancel{\frac{t^{10}}{30} + C} = \frac{(x^3+2)^{10}}{30} + C$

2.21

a) $\int \frac{e^x}{1+e^x}\, dx = \left[\begin{array}{l} t = e^x \Rightarrow x = g(t) = \ln t \\ \frac{dx}{dt} = \frac{1}{t} \Rightarrow dx = \frac{1}{t}dt \end{array}\right]$

$= \int \frac{t}{1+t} \cdot \frac{1}{t} dt = \int \frac{1}{1+t}\, dt = \ln|1+t| + C = \ln(1+e^x) + C$

b) $\int \frac{x}{\sqrt{x^2+1}}\, dx = \left[\begin{array}{l} t = x^2+1 \Rightarrow x = \sqrt{t-1}, \quad t>1 \\ \frac{dx}{dt} = \frac{1}{2}\frac{1}{\sqrt{t-1}}, \; dx = \frac{1}{2\sqrt{t-1}}\, dt \end{array}\right]$

$\int \frac{\cancel{\sqrt{t-1}}}{\sqrt{t}} \cdot \frac{1}{2} \cdot \frac{1}{\cancel{\sqrt{t-1}}}\, dt = \int \frac{1}{2\sqrt{t}}\, dt = t^{1/2} + C = \sqrt{x^2+1} + C$

(2.22)

a) $\int_0^2 2x\sqrt{4-x^2}\,dx = \left[\begin{array}{l} t = 4-x^2 \Rightarrow x = g(t) = \sqrt{4-t}, \quad t<4 \\ dx = g'(t)dt = -\frac{1}{2}\frac{1}{\sqrt{4-t}}\,dt \end{array}\right]$

$$\int_{t=4-0^2=4}^{t=4-2^2=0} \cancel{2\sqrt{4-t}}\,\sqrt{t}\cdot\left(-\frac{1}{\cancel{2}}\right)\frac{1}{\cancel{\sqrt{4-t}}}\,dt = \int_4^0 -t^{1/2}\,dt = \int_0^4 t^{1/2}\,dt$$

$$= \left[2t^{3/2}/3\right]_0^4 = \left(\frac{2\cdot 4^{3/2}}{3}\right) - (0) = \frac{2\cdot 8}{3} = \frac{16}{3}$$

MINNESREGEL

$$\boxed{\begin{array}{l} x = g(t), \quad \frac{dx}{dt} = g'(t) \\ dx = g'(t)\,dt \quad \boxed{F(x)dx = F(g(t))\cdot g'(t)dt} \end{array}}$$

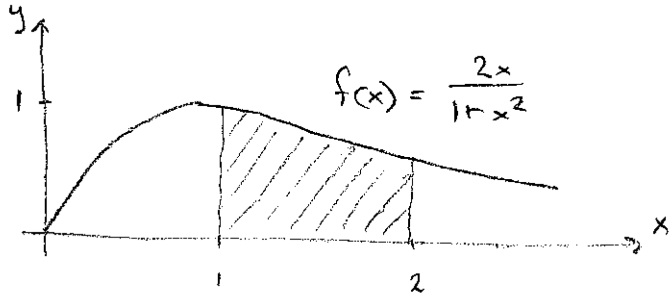
b) $\int_0^{\pi/2} 2\cos x\,(1+\sin x)\,dx = \left[\begin{array}{l} t = 1+\sin x \Rightarrow x = g(t) = \arcsin(t-1) \\ dx = g'(t)dt = \frac{1}{\sqrt{-t^2+2t}}\,dt \end{array}\right]$

$$\boxed{D(\arcsin(t-1)) = \frac{1}{D(1+\sin x)}} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-\sin^2 x}} =$$

$$= \frac{1}{\sqrt{1-(t-1)^2}} = \frac{1}{\sqrt{1-(t^2-2t+1)}} = \frac{1}{\sqrt{1-t^2+2t-1}} = \frac{1}{\sqrt{-t^2+2t}}$$

$$\int_1^2 2\sqrt{-t^2+2t}\cdot t\cdot\frac{1}{\sqrt{-t^2+2t}}\,dt = \int_1^2 2t\,dt = \left[t^2\right]_1^2 = (2^2)-(1^2) = 3$$

(2,23) Grafen till funktionen $f(x) = 2x/(1+x^2)$ är ritad i figuren nedan. Beräkna den markerade arean exakt.




$$\int_1^2 \frac{2x}{1+x^2}\,dx = \left[\begin{array}{l} t = 1+x^2 \Rightarrow x = g(t) = \sqrt{t-1},\ t>1 \\ dx = g'(t)\,dt = \frac{1}{2}\cdot\frac{1}{\sqrt{t-1}}\,dt \end{array}\right] = \quad \begin{array}{l} x=2 \Rightarrow t=5 \\ x=1 \Rightarrow t=2 \end{array}$$

$$\int_2^5 \frac{\cancel{2}\,\cancel{\sqrt{t-1}}}{t}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\cancel{\sqrt{t-1}}}\,dt = \int_2^5 \frac{1}{t}\,dt = \Big[\ln|t|\Big]_2^5 = \ln 5 - \ln 2 = \ln\frac{5}{2}$$

(2.24)

a)
$$\int_0^{1/2} \frac{4x}{\sqrt{1-x^2}}\,dx = \left[\begin{array}{l} t = 1-x^2 \Rightarrow x = g(t) = \sqrt{1-t},\ t<1 \\ dx = g'(t)\,dt = -\frac{1}{2}\frac{1}{\sqrt{1-t}}\,dt \end{array}\right] = \left(\begin{array}{l} x=\frac{1}{2} \to t = \frac{3}{4} \\ x=0 \to t=1 \end{array}\right)$$

$$= \int_1^{3/4} \frac{4\,\cancel{\sqrt{1-t}}}{\sqrt{t}}\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{\cancel{\sqrt{1-t}}}\,dt = -2\int_1^{3/4} t^{-1/2}\,dt = 2\int_{3/4}^{1} t^{-1/2}\,dt =$$

$$= 2\Big[2t^{1/2}\Big]_{3/4}^{1} = 2(2) - 2\left(2\left(\tfrac{3}{4}\right)^{1/2}\right) = 4 - 2\sqrt{3}$$

b)
$$\int_1^e \frac{(1+\ln x)^2}{x} = \left[\begin{array}{l} t = 1+\ln x \Rightarrow x = g(t) = e^{t-1},\ t>1 \\ dx = g'(t)\,dt = e^{t-1}\cdot dt \end{array}\right] \quad \begin{array}{l} x=e \to t=2 \\ x=1 \to t=1 \end{array}$$

$$\int_1^2 \frac{t^2}{\cancel{e^{t-1}}}\cdot\cancel{e^{t-1}}\,dt = \int_1^2 t^2\,dt = \left[\frac{t^3}{3}\right]_1^2 = \frac{2^3}{3} - \frac{1^3}{3} = \frac{7}{3}$$

(2,25) Bestäm $\int \frac{dx}{x \ln x}$ genom att substituera $t = \ln x$

$$\int \frac{1}{x \ln x}\, dx = \begin{bmatrix} t = \ln x & \Rightarrow & x = g(t) = e^t \\ dx = g'(t)\,dt = e^t\,dt \end{bmatrix}$$

$$= \int \frac{1}{\cancel{e^t} \cdot t} \cdot \cancel{e^t}\, dt = \int \frac{1}{t}\, dt = \ln|t| + C = \ln(\ln x) + C$$

(2,26) a) $$\int \frac{\sin x}{\cos^2 x}\, dx = \begin{bmatrix} t = \sin x & \Rightarrow & x = g(t) = \arcsin t \\ dx = g'(t)\,dt = \frac{1}{\sqrt{1-t^2}}\,dt \end{bmatrix}$$

$$\left[ D(\arcsin t) = \frac{1}{D \sin x} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-\sin^2 x}} = \frac{1}{\sqrt{1-t^2}} \right]$$

$$= \int \frac{t}{\sqrt{1-t^2}^{\,3}}\, dt = \int \frac{t}{(1-t^2)^{3/2}}\, dt = \begin{bmatrix} y = 1-t^2 & \Rightarrow & t = g(y) = \sqrt{1-y} \\ dt = g'(y)\,dy = -\frac{1}{2}\frac{1}{\sqrt{1-y}}\cdot dy \end{bmatrix}$$

$$= \int \frac{\cancel{\sqrt{1-y}}}{y^{3/2}} \cdot \left(-\frac{1}{2}\right) \frac{1}{\cancel{\sqrt{1-y}}}\, dy = -\frac{1}{2}\int y^{-3/2}\, dy = y^{-1/2} = \frac{1}{\sqrt{1-t^2}} = \frac{1}{\sqrt{1-\sin^2 x}} + C$$

$$= \frac{1}{\sqrt{\cos^2 x}} + C = \frac{1}{\cos x} + C$$

**Alt:** $$\begin{bmatrix} t = \cos x & \Rightarrow & x = g(t) = \arccos x \\ dx = g'(t)\,dt = -\frac{1}{\sqrt{1-t^2}}\,dt \end{bmatrix} = \int \frac{\sqrt{1-t^2}}{t^2} \cdot \frac{-1}{\sqrt{1-t^2}}\, dt = -\int t^{-2}\, dt =$$

$$= t^{-1} + C = \frac{1}{\cos x} + C$$

(2.27) Bestäm

$$\int \sin^2 x \, dx$$

genom att skriva om identiteten. $\boxed{\cos 2x = 1 - 2\sin^2 x}$

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Alt 1. $\int \sin^2 x \, dx = \int \left(\frac{1}{2} - \frac{1}{2}\cos 2x\right) dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

Alt -. $\int \sin^2 x \, dx = \left[\begin{array}{l} t = \sin x \Rightarrow x = g(t) = \arcsin t \\ dx = g'(t)\,dt = \frac{1}{\sqrt{1-t^2}}dt \end{array}\right.$

$$\left( D(\arcsin t) = \frac{1}{D(\sin x)} = \frac{1}{\cos x} = \frac{1}{\sqrt{1-\sin^2 x}} = \frac{1}{\sqrt{1-t^2}} \right)$$

$$\int \sin^2 x \, dx = \int t^2 \cdot \frac{1}{\sqrt{1-t^2}} dt = \left[\begin{array}{l} y = 1-t^2 \Rightarrow t = g(y) = \sqrt{1-y} \\ dt = g'(y)\,dy = -\frac{1}{2} \cdot \frac{1}{\sqrt{1-y}} dy \end{array}\right]$$

$$= \int (1-y) \cdot \frac{1}{\sqrt{y}} \cdot \left(-\frac{1}{2}\right) \cdot \frac{1}{\sqrt{1-y}} dy = \int \left(-\frac{1}{2}\right) \cdot \frac{\sqrt{1-y}}{\sqrt{y}} dy = \left[\begin{array}{l} z = 1-y \Rightarrow y = g(z) = 1-z \\ dy = g'(z)\,dz = -dz \end{array}\right]$$

$$= \int -\frac{1}{2} \cdot \frac{\sqrt{z}}{\sqrt{1-z}} \cdot -dz = \int \frac{1}{2} \frac{\sqrt{z}}{\sqrt{1-z}} dz$$

... jag kommer inte <u>framåt</u>!

(2.28) Beräkna integralen $\displaystyle\int_0^1 \frac{1}{e^x+e^{-x}}\,dx = \left[\begin{array}{l} t = e^x \Rightarrow x = g(t) = \ln t,\ t>1 \\ dx = g'(t)\,dt = \frac{1}{t}\,dt \end{array}\right.$

$$= \int_1^e \frac{1}{t+\frac{1}{t}}\cdot\frac{1}{t}\,dt = \int_1^e \frac{1}{t^2+1}\,dt = \Big[\arctan t\Big]_1^e = \underline{\underline{\arctan e - \frac{\pi}{4}}} \quad + C$$

$$\frac{1}{t^2+1} = D(\arctan t)$$

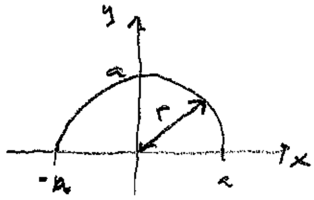
HUR SKALL VI INSE DETTA ?

(2.29) ?

$f(x) = \sqrt{a^2-x^2}$ , $-a \le x \le a$, $a \geq 0$

a) Visa med Pythagoras sats att kurvan är en halvcirkelbåge

$\boxed{x^2+y^2=r^2}$




$y = f(x)$

VL: $x^2 + (\sqrt{a^2-x^2})^2 = r^2$

$x^2 + a^2 - x^2 = r^2$

$a^2 = r^2 \Rightarrow \underline{\underline{r=a}}$ vsv.

b) Räkna ut arean av halvcirkeln genom att beräkna

$$\int_{-a}^{a} \sqrt{a^2-x^2}\,dx = \left[\begin{array}{l} t = a^2-x^2 \Rightarrow x = g(t) = \sqrt{a^2-t}\ ,\ t>0 \\ dx = g'(t)\,dt = -\frac{1}{2}\frac{1}{\sqrt{a^2-t}}\cdot dt \end{array}\right] \leftarrow ?$$

$\int$

**2.29** $y = f(x) = \sqrt{a^2 - x^2}$ , $a \le x \le a$ , $a \geq 0$

a) $\underline{r^2} = x^2 + y^2 = x^2 + \sqrt{a^2 - x^2}^{\,2} = x^2 + a^2 - x^2 = \underline{a^2}$ (oberoende av $x$ och $y$)

b) $$\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx = \left[ \begin{array}{l} x = a\sin t \Rightarrow dx = a\cos t\, dt \\ t_0 = -\pi/2 \quad t_1 = \pi/2 \end{array} \right]$$

$$\int_{-\pi/2}^{\pi/2} \sqrt{a^2 - a^2\sin^2 t} \cdot a\cos t\, dt = a^2 \int_{-\pi/2}^{\pi/2} \sqrt{1 - \sin^2 t}\ \cos t\, dt = a^2 \int_{-\pi/2}^{\pi/2} \cos^2 t\, dt =$$

$$= a^2 \int_{-\pi/2}^{\pi/2} \left(\frac{1}{2} + \frac{1}{2}\cos 2t\right) dt = \left[ \frac{a^2 t}{2} + \frac{a^2}{4} \cdot \sin 2t \right]_{-\pi/2}^{\pi/2} = \left( \frac{a^2\pi}{4} + \frac{a^2}{4} \cdot \underbrace{\sin\pi}_{=0} \right) - \left( \frac{a^2(-\pi)}{4} + \frac{a^2}{4} \underbrace{\sin(-\pi)}_{=0} \right)$$

$$= \frac{\pi a^2}{4} + \frac{\pi a^2}{4} = \frac{2\pi a^2}{4} = \frac{\pi a^2}{2}$$

En halv cirkelskiva

**2.30** $\int \sqrt{\frac{1-x}{1+x}} \cdot dx$ , $-1 < x < 1$

Ledning: Förläng bråket med $(1-x)$

$$\int \sqrt{\frac{(1-x)(1-x)}{(1+x)(1-x)}}\, dx = \int \sqrt{\frac{(1-x)^2}{1-x^2}}\, dx = \int \frac{1-x}{\sqrt{1-x^2}} = \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{x}{\sqrt{1-x^2}} dx =$$

$$\left[ \begin{array}{l} t = 1 - x^2 \Rightarrow x = g(t) = \sqrt{1-t} \\ dx = g'(t)\, dt = -\frac{1}{2} \frac{1}{\sqrt{1-t}} dt \end{array} \right] = \arcsin x - \int \frac{\cancel{\sqrt{1-t}}}{\sqrt{t}} \cdot \left(-\frac{1}{2}\right) \frac{1}{\cancel{\sqrt{1-t}}} dt =$$

$$\arcsin x + \frac{1}{2} \int t^{-1/2}\, dt = \arcsin x + t^{1/2} + C = \arccos x + \sqrt{1-x^2} + C$$

# Partiell integration

Lektion 16

$$\boxed{\int f(x) \cdot g(x) \cdot dx = f(x)\, G(x) - \int f'(x)\, G(x)\, dx}$$

$$\int f(x)\,g(x)\,dx = f(x)\,G(x) - \int f'(x)\,G(x)\,dx$$

**2.31** Bestäm

a) $\int x \cos x \, dx =$

$= x \sin x - \int \sin x \, dx = x \sin x + \cos x + C$

b) $\int x(\sin x - e^x) = x(-\cos x - e^x) - \int (-\cos x - e^x)\,dx =$

$-x(\cos x + e^x) + \int (\cos x + e^x)\,dx = -x(\cos x + e^x) + \sin x + e^x + C$

c) $\int x \cdot e^{-2x} \cdot dx = \dfrac{x \cdot e^{-2x}}{-2} - \int \dfrac{e^{-2x}}{-2}\,dx = -\dfrac{x e^{-2x}}{2} + \dfrac{1}{2}\int e^{-2x}\,dx$

$= -\dfrac{x e^{-2x}}{2} + \dfrac{1}{2}\cdot\dfrac{e^{-2x}}{(-2)} + C = e^{-2x}\left(-\dfrac{x}{2} - \dfrac{1}{4}\right) + C = \dfrac{-e^{-2x}(2x+1)}{4} + C$

d) $\int x \cos\frac{x}{2}\,dx = 2 \cdot x \sin\frac{x}{2} - \int 2 \sin\frac{x}{2}\,dx = 2x\sin\frac{x}{2} + 4\cos\frac{x}{2} + C$

e) $\int x^3 \cdot \ln x \, dx = \ln x \cdot \dfrac{x^4}{4} - \int \dfrac{1}{x}\cdot\dfrac{x^4}{4}\,dx = \dfrac{x^4}{4}\ln x - \dfrac{x^4}{16} + C$

f) $\int (1+x)\cdot e^{-x} = -(1+x)\,e^{-x} - \int -e^{-x}\,dx = -e^{-x}(1+x) - e^{-x} + C$

$= -e^{-x}(2+x) + C$

**2.32** Bestäm $\int x^2 \cdot e^x\,dx = x^2 e^x - \int 2x e^x\,dx =$

$x^2 e^x - 2\int x e^x\,dx = x^2 e^x - 2\left(x e^x - \int e^x\,dx\right) =$

$x^2 e^x - 2x e^x + 2e^x + C = e^x(x^2 - 2x + 2) + C$

2.33 Ange samtliga primitiva funktioner till $f(x) = \frac{1}{\sqrt{x}} \cdot \ln x$

$$\int f(x)\,dx = \int \frac{1}{\sqrt{x}} \ln x\,dx = \int x^{-1/2} \ln x\,dx = \int (\ln x) \cdot (x^{-1/2})\,dx$$

$$= \ln x \cdot x^{1/2} \cdot 2 - \int \left(\frac{1}{x} \cdot x^{1/2} \cdot 2\right) dx = 2\sqrt{x} \ln x - 2\int x^{-1/2} dx =$$

$$= 2\sqrt{x} \ln x - 2\, x^{1/2} \cdot 2 + C = 2\sqrt{x}(\ln x - 2) + C$$

2.34 Beräkna

a) $$\int_0^1 x e^{-3x}\,dx = \left[\frac{x\, e^{-3x}}{-3}\right]_0^1 - \int_0^1 \frac{e^{-3x}}{-3}\,dx =$$

$$= \left[-\frac{x e^{-3x}}{3} - \frac{e^{-3x}}{9}\right]_0^1 = \left(-\frac{e^{-3}}{3} - \frac{e^{-3}}{9}\right) - \left(0 - \frac{1}{9}\right) = \frac{1 - 4e^{-3}}{9}$$

b) $$\int_0^{\pi} x \sin x = \left[-x \cos x\right]_0^{\pi} + \int_0^{\pi} \cos x\,dx =$$

$$\left[-x \cos x + \sin x\right]_0^{\pi} = (-\pi \cdot \underbrace{\cos \pi}_{-1} + \underbrace{\sin \pi}_{0}) - \underbrace{(0 + \sin 0)}_{0} = \pi$$

(2.35) Bestäm $\int (1-x)^2 \cdot e^x \, dx = (1-x)^2 e^x - \int -2(1-x) e^x \, dx$

$$= (1-x)^2 e^x + 2\left((1-x)e^x - \int (-1) e^x \, dx\right) = (1-x)^2 e^x + 2(1-x)e^x + 2e^x + C$$

$$= e^x\left(1 - 2x + x^2 + 2 - 2x + 2\right) + C = e^x\left(x^2 - 4x + 5\right) + C$$

(2.37) $f(x) = \ln x$

$$\int f(x)\,dx = \int 1 \cdot \ln x \, dx = x \ln x - \int \frac{1}{x} \cdot x \, dx =$$

$$= x \ln x - \int dx = x \ln x - x + C = x(\ln x - 1) + C$$

# Partialbråksuppdelning

## 2.41 Skriv på ett bråkstreck

a) $$\frac{5}{x} + \frac{2}{x+1} = \frac{5(x+1) + 2x}{x(x+1)} = \frac{5x+5+2x}{x(x+1)} = \frac{3x+5}{x(x+1)}$$

b) $$\frac{6}{x-1} - \frac{2}{x+1} = \frac{6(x+1) - 2(x-1)}{(x+1)(x-1)} = \frac{6x+6-2x+2}{(x+1)(x-1)} = \frac{4x+8}{(x+1)(x-1)} = \frac{4(x+2)}{(x+1)(x-1)}$$

## 2.42 Dela upp polynomen i faktorer

a) $$p(x) = x^2+3x+2 = (x+1)(x+2)$$

$$\begin{array}{l} x+1 \\ \hline x^2+3x+2 \;\big|\; x+2 \\ -(x^2+2x) \\ \hline 0 \quad 1x+2 \\ -(x+2) \\ \hline 0 \end{array}$$

b) $$p(x) = 3x^2-30x+72 =$$
$$= 3(x^2-10x+24) =$$
$$= 3(x-4)(x-6)$$

c) $$p(x) = x^2-6x = x(x-6)$$

d) $$p(x) = x^2-10x+25 = (x+5)^2$$

e) $$p(x) = 2x^2-x-1 = x(2x-1)-1$$

$$= (x-1)(2x+1)$$

$$\begin{array}{l} x \\ \hline 2x^2-x-1 \;\big|\; 2x-1 \\ -(2x-x) \\ \hline 0 \quad 0-1 \end{array}$$

$$\begin{array}{l} x-1 \\ \hline 2x-x-1 \;\big|\; 2x+1 \\ -(2x+x) \\ \hline -2x-1 \\ -(-2x-1) \\ \hline 0 \end{array}$$

f) $$p(x) = x^3+x^2-2x = x(x^2-x-2)$$
$$= x(x-2)(x+1)$$

2.43 Partialbråksuppdela de rationella funktionerna

a) $f(x) = \dfrac{7x+12}{x^2+3x+2} = \dfrac{7x+12}{(x+1)(x+2)} = \dfrac{a}{(x+1)} + \dfrac{b}{(x+2)} =$

$$a(x+2) + b(x+1) \equiv 7x+12$$

$$x(a+b) + (2a+b) \equiv 7x+12$$

$$\begin{cases} a+b = 7 & 1) \\ 2a+b = 12 & 2) \end{cases}$$

$2) - 1) \Rightarrow \quad 2a+b-(a+b) = 12-7$

$a = 5$

$b = 2$

Svar: $f(x) = \dfrac{5}{x+1} + \dfrac{2}{x+2}$

b) $f(x) = \dfrac{6}{3x^2-30x+72} = \dfrac{6}{3(x^2-10x+24)} = \dfrac{2}{(x-4)(x-6)} =$

$\dfrac{a}{(x-4)} + \dfrac{b}{(x-6)} \Rightarrow \quad a(x-6)+b(x-4) \equiv 2$

$x(a+b) + (-6a-4b) \equiv 0 \cdot x + 2$

$$\begin{cases} a+b = 0 & 1)\ a = -b \\ -6a-4b = 2 & 2) \end{cases}$$

instoppat i 2)

$-6(-b) - 4b = 2$

$2b = 2$

$b = 1 \quad a = -1$

Svar: $f(x) = \dfrac{1}{x-6} - \dfrac{1}{x-4}$

2,43 Partialbråksuppdela de rationella funktionerna

c) $$\frac{12}{x^2-6x} = \frac{12}{x(x-6)} = \left[\frac{a}{x} + \frac{b}{x-6}\right] = \frac{a(x-6)+b(x)}{x(x-6)}$$

$$\Rightarrow \quad a(x-6)+b\cdot x \equiv 12$$

$$x(a+b) - 6\cdot(a) \equiv 12 \Rightarrow \begin{cases} a+b=0 \\ -6a=12 \end{cases} \quad \begin{cases} a=-2 \\ b=2 \end{cases}$$

Svar: $$\frac{12}{x^2-6x} = \frac{2}{x-6} - \frac{2}{x}$$

d) $$\frac{3}{2x^2-x-1} = \frac{3}{(2x+1)(x-1)} = \frac{a}{(2x+1)} + \frac{b}{(x-1)} = \frac{a(x-1)+b(2x+1)}{(2x+1)(x-1)}$$

$$a(x-1)+b(2x+1) \equiv 3$$

$$x(a+2b) + (b-a) \equiv 3 \Rightarrow \begin{cases} a+2b=0 \\ b-a=3 \end{cases}$$

$$3b = 3$$

$$b=1 \Rightarrow a=-2$$

Svar: $$\frac{3}{2x^2-x-1} = \frac{1}{(x-1)} - \frac{2}{(2x+1)}$$

2,44 a) Lös de obestående integralerna

$$\int \frac{7x+12}{x^2+3x+2}\,dx = \int \frac{7x+12}{(x+1)(x+2)} = \int\left(\frac{a}{x+1} + \frac{b}{x+2}\right)dx =$$

$$a(x+2)+b(x+1) \equiv 7x+12$$

$$x(a+b) + (2a+b) \equiv 7x+12 \Rightarrow \begin{cases} a+b=7 \\ 2a+b=12 \end{cases}$$

$$a = 12-7 = 5, \quad b=2$$

$$= \int\left(\frac{5}{x+1} + \frac{2}{x+2}\right)dx = 5\ln|x+1| + 2|x+2| + C$$

2,44

b) $\int \frac{12}{x^2-6x}\,dx = \int \frac{12}{x(x-6)}\,dx = \int\left(\frac{a}{x}+\frac{b}{x-6}\right)dx$

$a(x-6)+b(x) \equiv 12$

$x(a+b)-6a \equiv 12$ $\quad \begin{cases} a+b=0 \\ -6a=12 \end{cases} \Rightarrow a=-2 \quad b=2$

$= \int\left(\frac{2}{x-6}-\frac{2}{x}\right)dx = 2\ln(x-6) - 2\ln x + C$

c) $\int \frac{3}{2x^2-x-1}\cdot dx = \int \frac{3}{(2x+1)(x-1)}\,dx =$ (uppg 2,43)

$= \int\left(\frac{1}{(x-1)} - \frac{2}{(2x+1)}\right)dx = \ln(x-1) - \frac{2}{2}\ln(2x+1) + C$

2,45 Bestäm en primitiv funktion till

a) $f(x) = \frac{3x+1}{x} = 3+\frac{1}{x}$

$F(x) = 3x + \ln x$

b) $f(x) = \frac{x^2+2x-1}{x^2-1} = \frac{x^2+2x-1}{(x+1)(x-1)} =$

$\begin{array}{l} \quad 1 \\ x^2+2x-1 \;\big|\; x^2-1 \\ -(x^2-0-1) \\ \quad 2x \end{array}$

$\begin{array}{l} \quad x-3 \\ x^2+2x-1 \;\big|\; x-1 \\ -(x^2-x) \\ \quad 3x-1 \\ \quad -3x-3 \\ \quad\quad 2 \end{array}$

$\begin{array}{l} \quad x+1 \\ x^2+2x-1 \;\big|\; x+1 \\ (x^2+x) \\ \quad x-1 \\ \quad -x+1 \\ \quad\quad 2 \end{array}$

$= 1 + \frac{2x}{x^2-1} = 1 + \frac{a}{(x+1)} + \frac{b}{(x-1)}$

$= 1 + \frac{a(x-1)}{\cdots} + \frac{b(x+1)}{\cdots} \Rightarrow \begin{cases} a(x-1)+b(x+1) = 2x \\ x(a+b)+(b-a) = 2x \end{cases} \quad \begin{cases} a+b=2 \\ b-a=0 \end{cases}$

$b=1 \quad a=+1$

forts.

$$f(x) = 1 + \frac{1}{(x-1)} + \frac{1}{(x+1)}$$

$$F(x) = x + \ln(x-1) + \ln(x+1)$$

$$F(x) = x + \ln(x^2-1)$$

(2,46) Bestäm alla primitiva funktioner till

a) $f(x) = \dfrac{x^3-3x^2+x-2}{x-3} = x^2+1+\dfrac{1}{x-3}$

$$\begin{array}{l|l} x^2+1 & \\ \hline x^3-3x^2+x-2 & x-3 \\ -(x^3-3x^2) & \\ \hline 0 \quad 0 \quad x-2 & \\ \quad -(x-3) & \\ \hline \quad 1 & \end{array}$$

$$F(x) = \frac{x^3}{3} + x + \ln(x-3) + C$$

b) $f(x) = \dfrac{2x^2+2x+1}{x^2+x} =$

$$\left[\begin{array}{l|l} 2 & \\ \hline 2x^2+2x+1 & x^2+x \\ -(2x^2+2x) & \\ \hline 0+0+1 & \end{array}\right]$$

$$= 2 + \frac{1}{x^2+x} = 2 + \frac{a}{(x+1)} + \frac{b}{x} : \begin{cases} ax+b(x+1) \equiv 1 \\ x(a+b)+b \equiv 1 \end{cases}$$

$b=1 \quad a=-1$

$$= 2 + \frac{1}{x} - \frac{1}{x+1} \Rightarrow F(x) =$$

Bestäm alla primitiva funktioner till

2.46 a)

$$f(x) = \frac{x^3-3x^2+x-2}{x-3} = \left[\begin{array}{l|l} x^2+1 & \\ \hline x^3-3x^2+x-2 & x-3 \\ -(x^3-3x^2) & \\ \hline 0+0+x-2 & \\ \quad -(x-3) & \\ \hline \quad 0+1 & \end{array}\right] =$$

$$= x^2+1+\frac{1}{x-3}$$

$$\int f(x)\,dx = \int \left(x^2+1+\frac{1}{x-3}\right)dx = \frac{x^3}{3}+x+\ln(x-3)+C$$

b)

$$f(x) = \frac{2x^2+2x+1}{x^2+x} = \left[\begin{array}{l|l} 2 & \\ \hline 2x^2+2x+1 & x^2+x \\ (2x^2+2x) & \\ \hline 0+0+1 & \end{array}\right] = 2+\frac{1}{x^2+x} = 2+\frac{a}{x+1}+\frac{b}{x}$$

$$a(x)+b(x+1) \equiv 1 \quad \left\{\begin{array}{l} a+b=0, \; a=-1 \\ b=1 \end{array}\right\} = 2+\frac{1}{x}-\frac{1}{x+1}$$

$$\int f(x)\,dx = 2x+\ln x-\ln(x+1)+C$$

2.47 Bestäm den funktion $f(x)$ som har derivatan

$$f'(x) = \frac{2x^2-x}{x-2} \quad \text{och} \quad f(3)=20$$

Lösning:

$$f'(x) = \frac{2x^2-x}{x-2} = \left[\begin{array}{l|l} 2x+3 & \\ \hline 2x^2-x & x-2 \\ -(2x^2-4x) & \\ \hline 0+3x & \\ \quad -(3x-6) & \\ \hline \quad 0+6 & \end{array}\right] = 2x+3+\frac{6}{x-2}$$

$$\begin{cases} f(x) = \int f'(x)\,dx = x^2+3x+6\cdot\ln(x-2)+C \\ f(3)=20 \end{cases}$$

$$\Rightarrow 3^2+3\cdot 3+6\cdot\ln(3-2)+C=20$$

$$9+9+6\cdot 0+C=20$$

$$\underline{\underline{C=2}}$$

Svar: $f(x) = 6\ln(x-2)+x^2+3x+2$

**(2.48)** Beräkna integralen $\displaystyle\int_0^1 \frac{3x^4+11x^3+12x^2+4x+1}{x^2+3x+2}\,dx$

$$\left[\begin{array}{l|l} \multicolumn{1}{l}{\quad 3x^2+2x} & \\ \hline 3x^4+11x^3+12x^2+4x+1 & x^2+3x+2 \\ -(3x^4+9x^3+6x^2) & \\ \hline 0+2x^3+6x^2+4x & \\ \quad -(2x^3+6x^2+4x) & \\ \hline \quad 0 \quad 0 \quad 0 \quad +1 & \end{array}\right]$$

$$= \int_0^1 \left(3x^2+2x+\frac{1}{x^2+3x+2}\right)dx =$$

$$= \int_0^1 \left(3x^2+2x+\frac{1}{(x+2)(x+1)}\right)dx =$$

$$= \int_0^1 \left(3x^2+2x+\frac{a}{(x+2)}+\frac{b}{(x+1)}\right)dx =$$

$$\left[\begin{array}{l} a(x+1)+b(x+2) \equiv 1 \\ x(a+b)+a+2b \equiv 1 \\ \left.\begin{array}{l} a+b=0 \\ a+2b=1 \end{array}\right\} b=1,\ a=-1 \end{array}\right]$$

$$= \int_0^1 \left(3x^2+2x+\frac{1}{x+1}-\frac{1}{x+2}\right)dx$$

$$= \Big[x^3+x^2+\ln(x+1)-\ln(x+2)\Big]_0^1 =$$

$$\left(1^3+1^2+\ln(1+1)-\ln(1+2)\right)-\left(0^3+0^2+\ln(0+1)-\ln(0+2)\right) =$$

$$\int_0^1 f(x)\,dx = 2+\ln 2-\ln 3+\ln 2 = \underline{2+\ln(4/3)} \quad \text{eller} \quad \underline{2-\ln(3/4)}$$

**(2.49)** Lös de obestämda integralerna

a) $$\int \frac{x+2}{(x+1)^2}\,dx = \left[\frac{a}{x+1}+\frac{b}{(x+1)^2} \Rightarrow \begin{cases} a(x+1)+b \equiv x+2 \\ x\cdot(a)+(a+b) \equiv x+2 \end{cases} \quad \begin{cases} a=1 \\ a+b=2,\ b=1 \end{cases}\right]$$

$$= \int \left(\frac{1}{x+1}+\frac{1}{(x+1)^2}\right)dx = \ln(x+1)-\underline{\frac{1}{x+1}}+C$$

2.49

b) $\int \frac{x^2-2}{(x+3)^3}\,dx = \left| \frac{a}{x+3} + \frac{b}{(x+3)^2} + \frac{c}{(x+3)^3} \Rightarrow \right.$

$$\begin{cases} a(x+3)^2 + b(x+3) + c \equiv x^2-2 \\ x^2(a) + x(6a+b) + (9a+3b+c) = x^2-2 \end{cases}$$

$$\begin{cases} a = 1 \\ b = 0 - 6\cdot 1 = -6 \\ c = -2 - 9\cdot 1 - 3\cdot(-6) = 7 \end{cases}$$

$$= \int \frac{1}{x+3} - \frac{6}{(x+3)^2} + \frac{7}{(x+3)^3} = \ln(x+3) + \frac{6}{(x+3)} - \frac{7}{2(x+3)^2} + C$$

2.50 Partialbråksuppdela $\frac{x^2+1}{(x+1)(x+2)^2}$ genom att ansätta

$$\frac{x^2+1}{(x+1)(x+2)^2} \equiv \frac{a}{x+1} + \frac{b}{x+2} + \frac{c}{(x+2)^2} \Rightarrow$$

$$a(x+2)^2 + b(x+1)(x+2) + c(x+1) \equiv x^2+1$$

$$x^2(a+b) + x(4a+3b+c) + (4a+2b+c) \equiv x^2+1$$

$$\begin{aligned} a + b &= 1 &&\Rightarrow a = 1-b \\ 4a + 3b + c &= 0 &&\Rightarrow 4(1-b)+3b+c = 0 \Rightarrow c = b-4 \\ 4a + 2b + c &= 1 &&\Rightarrow 4(1-b)+2b+(b-4) = 1 \end{aligned}$$

$$-4b+2b+b = 1 + \cancel{4} - \cancel{4}$$

$$b = -1, \quad c = -5$$

$$a = 2$$

$$\Rightarrow \frac{x^2+1}{(x+1)(x+2)^2} = \frac{2}{(x+1)} - \frac{1}{(x+2)} - \frac{5}{(x+2)^2}$$

2.51 Lös den obestämda integralen $\int \frac{4(x-1)}{x^2(x-2)}\,dx$

$$= \left[ \frac{a}{x-2} + \frac{b}{x} + \frac{c}{x^2} = \frac{ax^2 + b(x-2)\cdot x + c(x-2)}{(x-2)x^2} \right]$$

$$\Rightarrow \quad ax^2 + bx^2 - 2bx + cx - 2c \equiv 4x - 4$$

$$x^2(a+b) + x(c-2b) - 2c \equiv 4x - 4$$

$$\left( \begin{array}{cccc} a & b & & = 0 \\ & -2b & c & = 4 \\ & & -2c & = -4 \end{array} \right) \quad \begin{array}{l} c = 2 \\ b = -1 \\ a = 1 \end{array}$$

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$$= \int \left( \frac{1}{x-2} - \frac{1}{x} - \frac{2}{x^2} \right) dx = 1\ln(x-2) - 1\ln(x) + \frac{2}{x} + C$$

Hitta den primitiva funktionen till $f(x) = \frac{1}{2x}$

Ledning: Skriv om $\ln(2x)$ med första logaritmlagen

$\ln 2x = \ln 2 + \ln x$ → $\ln 2$ kan behandlas som en konstant

$$\int f(x)\,dx = \int \frac{1}{2x} = \frac{\ln 2x}{2} + C_1 = \frac{\ln x}{2} + \underbrace{\ln 2 + C_1}$$

$$= \frac{\ln x}{2} + C$$

Rationella funktioner : $\frac{f(x)}{g(x)}$ , f(x), g(x) polynom

EXEMPEL.

$$\int \frac{x^4+3x^3+x+4}{x^3+3x^2-4}\,dx$$

Komplicerad

I) Om deg f(x) ≥ deg g(x) så utför vi polynomdivision

Kolla graden

4 deg

$$\begin{array}{l|l} \multicolumn{1}{l}{x \quad \text{kvot}} & \\ x^4+3x^3+0+x+4 & x^3+3x^2-4 \\ -(x^4+3x^3+0-4x) & \\ \hline \qquad\qquad 5x+4 \quad \text{rest} & \end{array}$$

$$\int \left( x + \frac{5x+4}{x^3+3x^2-4} \right) dx$$

OBS, VIKTIGT!
deg i täljare är större än deg i nämnare!

II) Faktorisera nämnaren så långt som möjligt

$$x^3+3x^2-4 = (x-1)(x^2+4x+4)$$

Gissa en rot $x=1$. Faktorsatsen ger att $(x-1)$ delar $x^3+3x^2-4$

$$\begin{array}{l|l} \multicolumn{1}{l}{x^2+4x+4} & \\ x^3+3x^2+0x-4 & x-1 \\ -(x^3-x^2) & \\ \hline \quad 4x^2+0x-4 & \\ -(4x^2-4x) & \\ \hline \quad 0 \quad 4x-4 & \\ \quad -\ 4x-4 & \\ \hline \quad 0 \quad 0 & \end{array}$$

Vi kollar om vi kan faktorisera $x^2+4x+4$

$$x^2+4x+4 = (x+2)^2 \quad \leftarrow \text{kvadreringsregeln}$$

$$\Rightarrow \quad x^3+3x^2-4 = (x-1)(x+2)^2$$

---

$$\int \frac{x^4+3x^3+x+4}{x^3+3x^2-4}\,dx = \int \left(x + \frac{5x+4}{(x-1)(x+2)^2}\right)dx$$

III) Partialbråksuppdelning

$$\frac{5x+4}{(x-1)(x+2)^2} = \left[\text{Ansätt } \frac{A}{x-1} + \frac{B}{x+2} + \frac{C}{(x+2)^2}\right] \qquad A, B, C \text{ är tal}$$

$$5x+4 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$$5x+4 \equiv A(x^2+4x+4) + B(x^2+x-2) + Cx - C$$

$$5x+4 \equiv (A+B)x^2 + (4A+B+C)x + (4A-2B-C)$$

$$\begin{pmatrix} A & B & 0 \\ 4A & B & C \\ 4A & -2B & -C \end{pmatrix} \begin{matrix} =0 \\ 5 \\ 4 \end{matrix} \Leftrightarrow \begin{pmatrix} A & B & \\ 8A & -B & \\ 4A & -2B & -C \end{pmatrix} \begin{matrix} =0 \\ =9 \\ =4 \end{matrix} \Leftrightarrow \begin{pmatrix} 9A \\ 8A-B \\ 4A-2B-C \end{pmatrix} \begin{matrix} =9 \\ =9 \\ =4 \end{matrix}$$

Gauss-elimination: $\Leftrightarrow \begin{cases} A = 1 \\ B = -1 \\ C = 2 \end{cases}$

IV

Det vill säga ...

$$\int \left(\frac{x^4+3x^3+x+4}{x^3+3x^2-4}\right)dx = \int \left(x + \frac{1}{x-1} - \frac{1}{x+2} + \frac{2}{(x+2)^2}\right)dx$$

$$= \frac{x^2}{2} + \ln|x-1| - \ln|x+2| - \frac{2}{x+2} + C$$

KLART!

Rationella funktioner : $\frac{f(x)}{g(x)}$ , $f(x)$ $g(x)$ polynom

EXEMPEL.

$$\int \frac{x^4+3x^3+x+4}{x^3+3x^2-4}\,dx$$ ← Komplicerad

I) Om $\underline{\deg}\ f(x) \geq \underline{\deg}\ g(x)$ så utför vi polynomdivision

$\boxed{\text{Kolla graden}}$ ← 4 deg "

kvot

$$\begin{array}{l}
\quad x \\
\hline
x^4+3x^3+0+x+4 \ \big|\ x^3+3x^2-4 \\
-(x^4+3x^3+0-4x) \\
\hline
\qquad 5x+4 \quad \text{rest}
\end{array}$$

$$\int\left(x + \frac{5x+4}{x^3+3x^2-4}\right)dx$$

OBS; VIKTIGT!,
deg i täljare är större än deg i nämnare !,

II) Faktorisera nämnaren så långt som möjligt

$$x^3+3x^2-4 = (x-1)(x^2+4x+4)$$

Gissa en rot $x=1$, Faktorsatsen ger att $(x-1)$ delar $x^3+3x^2-4$

$$\begin{array}{l}
\quad x^2+4x+4 \\
\hline
x^3+3x^2+0x-4 \ \big|\ x-1 \\
-(x^3-x^2) \\
\hline
\quad 4x^2+0x-4 \\
-(4x^2-4x\quad) \\
\quad \text{" " "}
\end{array}$$

Vi kollar om vi kan faktorisera $x^2+4x+4$

$$x^2+4x+4 = (x+2)^2 \quad \leftarrow \text{kvadreringsregeln}$$

$$\Rightarrow x^3+3x^2-4 = (x-1)(x+2)^2$$

$$\int \frac{x^4+3x^3+x+4}{x^3+3x^2-4}\,dx = \int\left(x + \frac{5x+4}{(x-1)(x+2)^2}\right)dx$$

III) Partialbråksuppdelning

$$\frac{5x+4}{(x-1)(x+2)^2} = \left[\ \overset{\text{Ansätt}}{\frac{A}{x-1}} + \frac{B}{x+2} + \frac{C}{(x+2)^2}\ \right]$$

A, B, C är tal

$$5x+4 \equiv A(x+2)^2 + B(x-1)(x+2) + C(x-1)$$

$$5x+4 \equiv A(x^2+4x+4) + B(x^2+x-2) + Cx - C$$

$$5x+4 \equiv (A+B)x^2 + (4A+B+C)x + (4A-2B-C)$$

$$\begin{pmatrix} A & B & 0 \\ 4A & B & C \\ 4A & -2B & -C \end{pmatrix}\begin{matrix} =0 \\ =5 \\ =4 \end{matrix} \Leftrightarrow \begin{pmatrix} A & B & \\ 8A & -B & \\ 4A & -2B & -C \end{pmatrix}\begin{matrix} =0 \\ =9 \\ =4 \end{matrix} \Leftrightarrow \begin{pmatrix} 9A \\ 8A-B \\ 4A-2B-C \end{pmatrix}\begin{matrix} =9 \\ =9 \\ =4 \end{matrix}$$

Gauss-elimination : $\Leftrightarrow \begin{cases} A=1 \\ B=-1 \\ C=2 \end{cases}$

IV

Det vill säga ...

$$\int\left(\frac{x^4+3x^3+x+4}{x^3+3x^2-4}\right)dx = \int\left(x + \frac{1}{x-1} - \frac{1}{x+2} + \frac{2}{(x+2)^2}\right)dx$$

$$= \frac{x^2}{2} + \ln|x-1| - \ln|x+2| - \frac{2}{\cdot\cdot} + C$$

KLART!

2.53 Partialbråksuppdela

a) $$\frac{x-1}{x(x^2+1)} = \left[ \frac{a}{x} + \frac{bx+c}{x^2+1} = \frac{a(x^2+1)+(bx+c)x}{x(x^2+1)} \right]$$

$$\left[ \begin{array}{l} x^2(a+b) + x(c) + a \equiv x-1 \\ \left( \begin{array}{cccc} a & b & & = 0 \\ & & c & = 1 \\ & & a & = -1 \end{array} \right) \Rightarrow b = 1 \end{array} \right]$$

$$= -\frac{1}{x} + \frac{x+1}{x^2+1}$$

b) $$\frac{3}{2x^3+2x} = \frac{3}{2x(x^2+1)} = \left[ \frac{a}{2x} + \frac{bx+c}{x^2+1} = \frac{a(x^2+1)+(bx+c)2x}{x(x^2+1)} \right]$$

$$a(x^2+1) + 2x(bx+c) \equiv 3$$

$$x^2(a+2b) + x(2c) + a \equiv 3$$

$$\left( \begin{array}{cccc} a & 2b & & = 0 \\ & & 2c & = 0 \\ & & a & = 3 \end{array} \right) \quad \begin{array}{l} b = -3/2 \\ c = 0 \end{array}$$

$$= \frac{3}{2x} - \frac{3x}{2(x^2+1)}$$

(2.54) Partialbråksoppdela

a) $$\frac{x^2+9}{(x+1)(x^2+4)} = \left[\overset{\text{Ansätt}}{\frac{a}{(x+1)} + \frac{bx+c}{(x^2+4)}}\right] = \frac{a(x^2+4)+(bx+c)(x+1)}{(x+1)(x^2+4)}$$

$$\Rightarrow \quad \cancel{ax^2}+4a+\cancel{bx^2}+\cancel{bx}+\cancel{cx}+c \equiv x^2+9$$

$$x^2(a+b)+x(b+c)+(4a+c) \equiv x^2+9$$

$$\left(\begin{array}{cccc} a & b & & =1 \\ & b & c & =0 \\ 4a & & c & =9 \end{array}\right) \qquad \begin{array}{ll} b=1-a & b=-1 \\ c=-b=a-1 & \cancel{-b}=1 \\ 4a+(a-1)=9 \Rightarrow a=2 & \end{array}$$

$$= \frac{2}{x+1} - \frac{x-1}{x^2+4} \quad \text{eller} \quad \frac{1-x}{x^2+4} + \frac{2}{x+1}$$

b) $$\frac{2x^2+x+2}{x(x^2+1)} = \left[\overset{\text{Ansätt}}{\frac{a}{x} + \frac{bx+c}{x^2+1}}\right] = \frac{a(x^2+1)+(bx+c)x}{x(x^2+1)}$$

$$\Rightarrow \quad ax^2+a+bx^2+cx \equiv 2x^2+x+2$$

$$x^2(a+b)+x\cdot(c)+(a) \equiv 2x^2+x+2$$

$$\left(\begin{array}{cccc} a & b & & =2 \\ & & c & =1 \\ a & & & =2 \end{array}\right) \qquad \begin{array}{l} b=0 \\ c=1 \\ \Rightarrow a=2 \end{array}$$

$$= \frac{2}{x} + \frac{1}{x^2+1}$$

2.55 Partialbråksuppdela

a) $$\frac{3x^2-1}{(x-1)^2(x^2+1)} = \left[ \text{Ansatt} \quad \frac{a}{(x-1)} + \frac{b}{(x-1)^2} + \frac{cx+d}{x^2+1} \right] = \frac{a(x-1)(x^2+1)+b(x^2+1)+(cx+d)(x-1)^2}{(x-1)^2(x^2+1)}$$

$$\Rightarrow \quad ax^3 - ax^2 + ax - a + bx^2 + b + (cx+d)(x^2-2x+1) \equiv 3x^2-1$$

$$x^3(a+b+c) + x^2(-a+b-2c+d) + x(a+c-2d) + (-a+b+d) \equiv 3x^2-1$$

$$\begin{array}{l} 1) \\ 2) \\ 3) \\ 4) \end{array} \begin{pmatrix} a & b & c & & = 0 \\ -a & b & -2c & d & = 3 \\ a & & c & -2d & = 0 \\ -a & b & & d & = -1 \end{pmatrix}$$

eliminera a

$$\begin{array}{ll} 5) & 1)+2) \\ 6) & 1)-3) \\ 7) & 1)+4) \end{array} \begin{pmatrix} 2b & -c & d & = 3 \\ b & & +2d & = 0 \\ 2b & +c & +d & = -1 \end{pmatrix}$$

eliminera b

$$\begin{array}{ll} 8) & 5)-2\cdot 6) \\ 9) & 5)-7) \end{array} \begin{pmatrix} -c & -3d & = 3 \\ -2c & & = 4 \end{pmatrix} \quad \Rightarrow 2-3d=3 \Rightarrow$$

$c = -2$

2

(2.55) Partialbråksuppdela

a)

$$\frac{3x^2-1}{(x-1)^2(x^2+1)} = \left[\text{Ansätt } \frac{a}{x-1} + \frac{b}{(x-1)^2} + \frac{cx+d}{x^2+1}\right] =$$

Partialbråksuppdela

2.55

b) $$\frac{2x(x+1)(x+7)}{(x-1)(x+3)(x^2+3)} = \left[\text{Ansätt } \frac{a}{(x-1)} + \frac{b}{(x+3)} + \frac{cx+d}{x^2+3}\right] =$$

$$a(x+3)(x^2+3) + b(x-1)(x^2+3) + (cx+d)(x-1)(x+3) \equiv 2x(x+1)(x+7)$$

$$\cancel{ax^3} + 3ax^2 + 3ax + 9a + \cancel{bx^3} - bx^2 + 3bx - 3b + \cancel{cx^3} + 2cx^2 - 3cx + dx^2 + 2dx - 3d \equiv \ldots$$

$$\begin{array}{l} 1)\ x^3; \\ 2)\ x^2; \\ 3)\ x; \\ 4)\ 1; \end{array} \left(\begin{array}{ccccc} a & b & c & & = 2 \\ 3a & -b & 2c & d & = 16 \\ 3a & 3b & -3c & 2d & = 14 \\ 9a & -3b & & -3d & = 0 \end{array}\right)$$

eliminera a

$$\begin{array}{l} 5) = 3\cdot1) - 2) \\ 6) = 3\cdot1) - 3) \\ 7) = 9\cdot1) - 4) \end{array} \left(\begin{array}{cccc} 4b & c & -d & = -10 \\ & 6c & -2d & = -8 \\ 12b & 9c & 3d & = 18 \end{array}\right)$$

eliminera b

Handpåläggning

$x = 1 \rightarrow$ $a = 2$

$x = -3 \rightarrow$ $b = -1$

$x = 0 \Rightarrow$ c försvinner och $d = 7$

Gausseliminering på $x^3$ termer ger

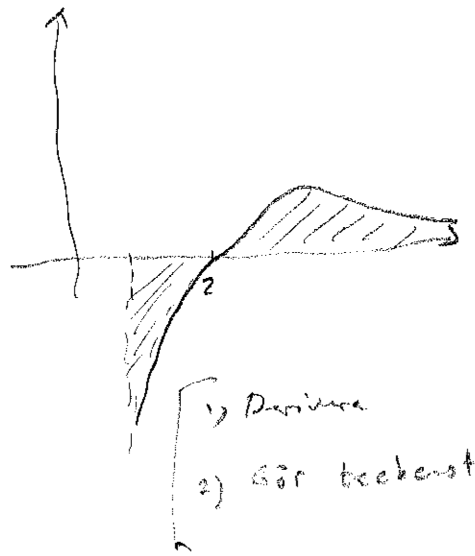
$$\begin{array}{cccc} a & b & c & = 2 \\ 2 & -1 & c & = 2 \\ & & c & = 2-2+1 = 1 \end{array}$$

$c = 1$

Generaliserade integraler Lektion 19

Ex: $x > 1$

$$\int_1^\infty \frac{\ln(x-1)}{x^2}\,dx$$




Farliga ställen $x = 1$

$x \to \infty$

1) Derivera

2) Gör teckenstudium

Idé! • Dela upp integralen i flera delar

• Bara en farlighet per del

$$\boxed{\int_1^\infty \frac{\ln(x-1)}{x^2}\,dx = \int_1^2 \frac{\ln(x-1)}{x^2}\,dx + \int_2^\infty \frac{\ln(x-1)}{x^2}\,dx}$$

produkt/[illegible]

partialbråks-uppdelning

$$\int \frac{\ln(x-1)}{x^2} = \int \frac{1}{x^2}\cdot\ln(x-1)\,dx = -\frac{1}{x}\ln(x-1) + \int \frac{1}{x}\cdot\frac{1}{(x-1)}\,dx$$

$$= -\frac{1}{x}\ln(x-1) + \int\left(\frac{1}{x-1} - \frac{1}{x}\right)dx = -\frac{1}{x}\ln(x-1) + \ln(x-1) - \ln x + C$$

(obs $x > 1 \Rightarrow$ inga absolutbelopp)

$$\int_2^\infty \frac{\ln(x-1)}{x^2}\,dx = \lim_{B\to\infty}\int_2^B \frac{\ln(x-1)}{x^2}\,dx = \Big[\qquad\qquad\Big]_2^B =$$

$$= \left(-\frac{1}{B}\ln(B-1) + \ln(B-1) - \ln B\right) - \left(0 + 0 - \ln 2\right) =$$

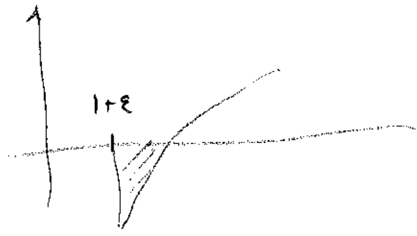
log

Standardgränsvärde

$$-\frac{\ln(B-1)}{B} + \ln\left(\frac{B-1}{B}\right) + \ln 2 = \underline{\underline{\ln 2}} \quad \text{konvergent}$$

0 potens →1

$$\int_1^2 \frac{\ln(x+1)}{x^2}\,dx$$




$$\lim_{\varepsilon \to 0} \int_{1+\varepsilon}^{2} \frac{\ln(x-1)}{x^2}\,dx = \left[-\frac{1}{x}\ln(x-1) + \ln(x-1) - \ln(x)\right]_{1+\varepsilon}^{2} =$$

ε = ett tal nära x=1

$$= \Big(-\frac{1}{2}\underbrace{\ln 1}_{0} + 0 - \ln 2\Big) - \Big(-\frac{\ln \varepsilon}{1+\varepsilon} + \ln \varepsilon - \ln(1+\varepsilon)\Big) =$$

$$= -\ln 2 + \underbrace{\Big(\frac{1}{1+\varepsilon} - 1\Big)}_{= \frac{1-(1+\varepsilon)}{1+\varepsilon} = \frac{-\varepsilon}{1+\varepsilon}} \ln \varepsilon + \ln(1+\varepsilon) =$$

$$= -\ln 2 - \underbrace{\frac{\varepsilon \ln \varepsilon}{1+\varepsilon}}_{\to 0} + \underbrace{\ln(1+\varepsilon)}_{\to \ln 1 = 0} \to -\ln 2 \quad \text{då} \quad \varepsilon \to 0^+$$

konvergent!

$$\int_1^{\infty} \frac{\ln(x+1)}{x^2}\,dx = -\ln 2 + \ln 2 = \underline{\underline{0}}$$

# Lektion 19

**2,70** Integralerna nedan är konvergenta. Beräkna deras värden

a) $$\int_2^\infty \frac{1}{x^2}\cdot dx = \lim_{B\to\infty}\int_2^B \frac{1}{x^2}\,dx = \lim_{B\to\infty}\left[-\frac{1}{x}\right]_2^B = \left(-\frac{1}{B}\right)-\left(-\frac{1}{2}\right) = \frac{1}{2}-\frac{1}{B}$$

$B\to\infty$

$$= \frac{1}{2}$$

b) $$\int_{10}^\infty x^{-10}\,dx = \lim_{B\to\infty}\int_{10}^B x^{-10}\,dx = \lim_{B\to\infty}\left[-\frac{x^{-9}}{9}\right]_{10}^B =$$

$$= \lim_{B\to\infty}\left(\left(-\frac{B^{-9}}{9}\right)-\left(-\frac{10^{-9}}{9}\right)\right) = \frac{10^{-9}}{9} = \frac{1}{9}\cdot 10^{-9} = \frac{10}{9}\cdot 10^{-10}$$

c) $$\int_1^\infty \frac{dx}{x\sqrt{x}} = \lim_{B\to\infty}\int_1^B \frac{1}{x\cdot x^{0,5}}\,dx = \lim_{B\to\infty}\int_1^B x^{-1,5}\,dx = \lim_{B\to\infty}\left[-2\,x^{-0,5}\right]_1^B =$$

$$= \lim_{B\to\infty}\left(\left(-\frac{2}{\sqrt{B}}\right)-\left(-\frac{2}{\sqrt{1}}\right)\right) = +2$$

$\frac{2}{\sqrt{B}} \to 0$

d) $$\int_8^\infty \frac{1}{x^{4/3}}\cdot dx = \lim_{B\to\infty}\int_8^B \frac{1}{x^{4/3}}\,dx = \lim_{B\to\infty}\left[-3\,x^{-1/3}\right]_8^B =$$

$$\lim_{B\to\infty}\left(\left(-3\,B^{-1/3}\right)-\left(-3\cdot 8^{-1/3}\right)\right) = \lim_{B\to\infty}\left(-\frac{3}{\sqrt[3]{B}}+\frac{3}{\sqrt[3]{8}}\right) = \frac{3}{\sqrt[3]{8}} = \frac{3}{2} = 1,5$$

$\frac{3}{\sqrt[3]{B}} \to 0$

**2.71** Undersök om integralerna konvergerar, Ange isåfall värdet.

a) $$\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \int_1^\infty x^{-0,5}$$ $\underline{0,5 < 1}$ → divergent ~~konvergent~~ !

enligt sats 2.25

b) $$\int_1^\infty \frac{1}{x^{-1,001}}\,dx = \int_1^\infty x^{1,001} = \int_1^\infty x^{-(-1,001)}$$ $-1,001 < 1$

enligt sats → $\underline{\text{divergent}}$ !

c) $$\int_1^\infty \frac{1}{x^{1,001}}\,dx = \int_1^\infty x^{-1,001}\,dx$$ $1,001 > 1$ Konvergent

$$= \lim_{B\to\infty} \int_1^B x^{-1,001}\,dx = \lim_{B\to\infty} \left[\left(\frac{x^{-0,001}}{-0,001}\right)\right]_1^B = \lim_{B\to\infty}\left[\underbrace{\left(\frac{B^{-0,001}}{-0,001}\right)}_{\downarrow\, 0} - \left(\frac{1^{-0,001}}{-0,001}\right)\right]$$

$$= \frac{1}{0,001} = \underline{\underline{1000}}$$

d) $$\int_0^1 \frac{1}{x^{1,001}}\,dx = \lim_{\varepsilon\to 0}\int_\varepsilon^1 \frac{1}{x^{1,001}}\,dx = \lim_{\varepsilon\to 0}\left[\frac{x^{-0,001}}{-0,001}\right]_\varepsilon^1 =$$

$$\lim_{\varepsilon\to 0}\left[\underbrace{\left(\frac{1^{-0,001}}{-0,001}\right)} - \left(\frac{\varepsilon^{-0,001}}{-0,001}\right)\right] = -1000 + \frac{1000}{\varepsilon^{0,001}} \longrightarrow \infty$$

$\varepsilon \to 0 \Rightarrow$

$\underline{\text{DIVERGENT}}$ !

Undersök om integralerna konvergerar. Ange isåfall värdet.

**2,72**

a) $\int_0^\infty e^{-x} = \lim_{B\to\infty} \int_0^B e^{-x}\,dx = \lim_{B\to\infty} \left[-e^{-x}\right]_0^B =$

$\lim_{B\to\infty} \left(\left(-e^{-B}\right) - \left(-e^{0}\right)\right) = 0 + 1 = 1$ <u>Konvergerar!</u>

b) $\int_1^\infty \frac{1}{x}\,dx = \int_1^\infty x^{-1}\,dx$ $\quad 1 \le 1$ vilket innebär att integralen divergerar!

$\boxed{\text{sats } 2{,}25}$

**2.73** Vilka av följande integraler är konvergenta? Avgör utan några beräkningar.

a) $\int_1^\infty x^{-0{,}3}\,dx$ $\left[\begin{array}{c} 0{,}3 \le 1 \\ \text{DIVERGENT} \end{array}\right]$

b) $\int_1^\infty x^{-1{,}5}\,dx$ $\left[\begin{array}{c} 1{,}5 > 1 \\ \text{KONVERGENT} \end{array}\right]$

c) $\int_1^\infty \frac{1}{\sqrt[3]{x}}\,dx = \int_1^\infty x^{-1/3}\,dx$ $\left[\begin{array}{c} 1/3 \le 1 \\ \text{DIVERGENT} \end{array}\right]$

d) $\int_1^\infty x^{3}\,dx = \int_1^\infty x^{-(-3)}\,dx$ $\left[\begin{array}{c} -3 \le 1 \\ \text{DIVERGENT} \end{array}\right]$

e) $\int_{1000}^\infty \frac{1}{x^5}\,dx = \int_{1000}^\infty x^{-5}\,dx = \left[\begin{array}{c} 5 > 1 \\ \text{KONVERGENT} \end{array}\right]$

f) $\int_1^\infty \frac{1}{\sqrt[5]{x^6}}\,dx = \int_1^\infty x^{-6/5}\,dx = \left[\begin{array}{c} 6/5 > 1 \\ \text{KONVERGENT} \end{array}\right]$